SCHEDULE ABEL SYMPOSIUM 2009
COMBINATORIAL ASPECTS OF
COMMUTATIVE ALGEBRA AND ALGEBRAIC GEOMETRY
JUNE 1-4, 2009, VOSS, NORWAY

Schedule

Sunday, May 31st.

Arrival
19.00-22.30 Dinner

Monday, June 1st.

07.00-10.00 Breakfast

10.00-10.45 J. Herzog: "Powers of componentwise linear ideals"
Coffee break
11.15-12.00 D. Maclagan: "Connected multigraded Hilbert schemes"

12.15 Lunch

15.00-15.45 D. Eisenbud: "Boij-Söderberg Theory: introduction and possible future perspectives"
16.00-16.45 M. Boij: "Hilbert functions and Betti numbers up to multiples and parameter spaces"
Coffee break
17.15-18.00 F.-O. Schreyer: "Boij-Söderberg theory for coherent sheaves on $\mathbb{P}^n$"

19.00 Dinner

Tuesday, June 2nd.

07.00-09.00 Breakfast

09.00-09.45 M. Stillman: "Green's conjecture and high rank syzygies on low rank quadrics"
10.00-10.45 J. Sidman: "Syzygies of the secant varieties of curves"
Coffee break
11.15-12.00 A. Conca: "Syzygies of Veronese and Koszul algebras"

12.15 Lunch

15.00-15.45 R. Thomas: "The Convex Hull of a Real Algebraic Variety"

16.00-16.45 A. Bertram: "Deconstructing Coherent Sheaves"
Coffee break
17.15-18.00 J. Kamnitzer: "Equivalences of derived categories from geometric sl(2) actions"

19.00 Dinner

Wednesday, June 3rd.

07.00-08.15 Breakfast

08.15-09.00 J. Weyman: "Quivers with potentials and their mutations"
09.10-09.55 A. Zelevinsky: "Cluster algebras via quivers with potentials"

10.15: Excursion. Meeting point outside the entrance of the hotel.

Around 18.00: return to the hotel
19.00 Dinner

Thursday, June 4th.

07.00-09.00 Breakfast

09.00-09.45 S. Payne: "Boundary complexes and weight filtrations"
10.00-10.45 S. Fomin: "Enumeration of plane curves and labeled floor diagrams"
Coffee break
11.15-12.00 E. Miller: "Applications of binomial commutative algebra"

12.15 Lunch

14.00-14.45 D. Laksov: "A formalism for equivariant cohomology"
15.00-15.45 A. S. Buch: "Quantum K-theory of Grassmannians"
Coffee break
16.15-17.00 W. Fulton: "Character formulas"

19.00 Dinner

Friday, June 5th.

07.00-10.00 Breakfast

Departure

Abstracts of talks

Jürgen Herzog: *Powers of componentwise linear ideals*

Let $S = K[x_1, \ldots, x_n]$ be the polynomial ring over a field K, and $I \subset S$ a graded ideal. It was proved by Cutkosky, Herzog, Trung and independently by Kodiyalam that the regularity of I^k is a linear function of k for large k. Of special interest in the case, when I has a linear resolution. Naively one would expect that for an ideal with linear resolution, all its powers have a linear resolution as well. But this is not the case. A first counterexample was given by Terai who observed that, if the characteristic of K is zero, then the Stanley–Reisner ideal of the natural triangulation of the real projective plane has a linear resolution, but the square does not. For ideals with linear resolution there exist criteria, among them the so-called x-condition that allow to test whether all of its powers have a linear resolution as well. For example, by using the x-condition, it was shown that all powers of a monomial ideal with linear resolution generated in degree 2 have linear resolutions. But this condition fails in many other cases, for example for the ideal of 2-minors of a generic symmetric 3×3-matrix are for most other ideals defining rational normal scrolls.

In this lecture I report on joint work with Hibi and Ohsugi. We consider componentwise linear ideal, that is, ideals for which each of its components generates an ideal with linear resolution. In characteristic 0 a componentwise linear is distinguished by the fact that its graded Betti numbers and that of its generic ideal coincide. Based on the theory of approximation complexes and results of Römer and Yanagawa, we are able to give a necessary and sufficient condition for a graded ideal to have the property that all its powers are componentwise linear. This criterion is applied to show that for certain classes of chordal graphs all powers of their vertex cover ideals are componentwise linear. We expect this to be true for any chordal graph.

Diane Maclagan: *Connected multigraded Hilbert schemes*

The multigraded Hilbert scheme, introduced by Haiman and Sturmfels, parameterizes all ideals in a polynomial ring with a fixed Hilbert function with respect to an abelian group grading. Unlike the usual Hilbert scheme of subschemes of projective space, multigraded Hilbert schemes are not always connected. I will discuss an approach to showing connectedness in special cases by walking along one-dimensional torus orbits. In particular, I will discuss joint work with Greg Smith where we apply these ideas to show that when the polynomial ring has two variables all multigraded Hilbert schemes are smooth and irreducible.

David Eisenbud: *Boij-Söderberg Theory: introduction and possible future perspectives*

Following a remarkable set of conjectures by Boij and Söderberg, there has been a flurry of research by Boij-Söderberg, Erman, Fløystad, Schreyer, Sam, Weyman, and myself with proofs and extensions of the basic ideas. The theory now comprises a number of results about the form of free resolutions of graded modules and the cohomology tables of sheaves. I'll try to explain the basic facts that have been uncovered, and indicate some open problems.

Mats Boij: *Hilbert functions and Betti numbers up to multiples and parameter spaces*

Since we now know the possible Betti tables of graded modules over the polynomial ring up to rational multiples, we can use this in order to study for examples parameter spaces of modules with a given Hilbert function. In particular, we can look at what happens when we start by a given Hilbert function H, and compare the paramter spaces for integral multiples, nH. The condition on the Hilbert function cuts the cone of Betti tables in a polytope, P, and we are lead to look at multiples of this polytope, nP.

We can also give conditions on the shape of the Betti tables, for example by restricting the degrees of the generators. Depending on this shape, we get different structures, for example regarding the existence of unique maximal or minimal Betti tables.

Frank-Olaf Schreyer: *Boij-Söderberg theory for coherent sheaves on* $\mathbb{P}^n$

Boij-Söderberg theory for vector bundles is a unique decomposition of their cohomology tables into a finite number of positive rational multiples of cohomology tables of supernatural vector bundles. The theory for coherent sheaves has a number new features: It is a decomposition into a unique **infinite** sum of tables of supernatural **sheaves** with possibly **real** coefficients. A number of questions are open. Eg.: Do we really get real coefficients? Correspond torsion free sheaves to series of vector bundles alone?

Michael Stillman: *Green's conjecture and high rank syzygies on low rank quadrics*

In the early 1980's, Mark Green noticed a remarkable (conjectured) relationship between subtle geometry of a smooth projective curve, and pieces of the free resolution of the ideal of the curve. In about 1990, David Eisenbud conjectured some possible extensions of Green's conjecture, stated in algebraic language. In this talk, we will first survey the basic concepts and conjectures, and then discuss constructions which provide counter-examples to some of these conjectures. There is still much to be understood in this fascinating story, and so we end with some open problems.

This is joint work with Hal Schenck.

Jessica Sidman: *Syzygies of the secant varieties of curves*

In the 1980's, work of Green and Lazarsfeld suggested that results on quadric generation of the ideal of a curve of high degree could be seen as a piece of a picture involving all of its higher syzygy modules. I will discuss some results regarding the syzygies of the secant variety of a high degree curve. We will see that a conjectural picture emerges for the syzygies of higher secant varieties which generalizes what we know to be true for curves.

This is joint work with Peter Vermeire.

Aldo Conca: *Syzygies of Veronese and Koszul algebras*

The goal of the talk is to present results about syzygies of Veronese rings obtained jointly with W.Bruns and T.Romer and results about syzygies of Koszul rings obtained jointly with L.Avramov and S.Iyengar. In particular, we show that the c-th Veronese ring of a polynomial ring over a field of characterisitic 0 satisfies the Green-Lazarsfeld property N_{c+1}. Moreover, we show that the the degrees of syzygies of a Koszul algebra grow at most as those of an algebra defined by monomials of degree 2.

Rekha Thomas: *The Convex Hull of a Real Algebraic Variety*

A central problem in many applications is to understand (compute or approximate) the convex hull of a real algebraic variety. For instance, minimizing a polynomial with real coefficients over $\mathbb{R}^n$ can be phrased as a linear program over the convex hull of a toric image of $\mathbb{R}^n$. Given an ideal over the reals, I will present a hierarchy of convex relaxations of the convex hull of the real variety of the ideal, called "theta bodies". This is inspired by a question posed by Lovasz to characterize those ideals for which the very first theta body equals the convex hull of the real variety. We solve this problem for zero-dimensional real radical ideals which covers the usual cases of interest in combinatorial optimization.

Joint work with Joao Gouveia and Pablo Parrilo

Aaron Bertram: *Deconstructing Coherent Sheaves*

To deconstruct is to analyze, in order to expose internal assumptions and subvert the apparent unity (courtesy of my desktop dictionary). Some of the assumptions about coherent sheaves that I'd like to subvert include: "ideal sheaves are always stable", and "non-trivial torsion subsheaves always destabilize a coherent sheaf". Our deconstruction, applied to the Hilbert scheme of points on the projective plane, will allow us to obtain new birational models that are combinatorial in nature, but also represent suitable moduli functors. This is work in progress with Izzet Coskun.

Joel Kamnitzer: *Equivalences of derived categories from geometric $sl(2)$ actions*

I will discuss a new technique for constructing equivalences of derived categories of coherent sheaves. This allows us to construct equivalences between the derived categories of cotangent bundles to Grassmannians. Our work generalizes the spherical twists of Seidel-Thomas and is based on work of Chuang-Rouquier. This is joint work with Sabin Cautis and Anthony Licata.

Jerzy Weyman *Quivers with potentials and their mutations*

This talk is based on a joint work with Harm Derksen and Andrei Zelevinsky. A quiver with potential (QP for short) is a quiver Q together with an element S of the path algebra of Q such that S is a linear combination of cyclic paths. We associate to S its Jacobian ideal, i.e. the two-sided ideal $J(S)$ in the path algebra KQ generated by the (noncommutative) partial derivatives of S with respect to the arrows of Q. The quotient $P(A,S)$ of the (completed) path algebra KQ by $J(S)$ is the Jacobian algebra associated to a QP. Such algebras appeared in physicists' work on superpotentials in the context of the Seiberg duality in mirror symmetry.

In this talk I will discuss the mutations for QPs and their (decorated) representations.

More precisely, this setup directly extends to QPs the Bernstein-Gelfand-Ponomarev reflection functors that played key role in quiver representations.

These mutations on the level of quivers coincide with the mutations appearing in the theory of cluster algebras.

In a related talk Andrei Zelevinsky will discuss the applications of QP's to cluster algebras.

Andrei Zelevinsky: *Cluster algebras via quivers with potentials*

Cluster algebras are commutative rings of a special kind making a surprising appearance in a variety of settings, including tilting theory, Poisson geometry, Teichmuller theory, representations of semisimple groups, etc. Their structure is governed by several piecewise-polynomial and rational recurrences on a regular tree. Although these recurrences are quite explicit and elementary, a direct proof of their conjectural properties seems to be hard to find. In a joint work with Harm Derksen and Jerzy Weyman we find their representation-theoretic interpretation in terms of quivers with potentials, that allows us to prove most of the conjectures in question. The theory of quivers with potentials and their representations will be addressed in Jerzy Weyman's talk at this Symposium. I will try to keep my talk reasonably self-contained.

Sam Payne: *Boundary complexes and weight filtrations*

The boundary complex of an algebraic variety is the dual complex of the boundary divisor in a compactification of a log resolution. I will present recent work showing that the homotopy type of this complex is independent of the choice of resolution and compactification, and give relations between these complexes and Deligne's weight filtration on singular cohomology.

Sergey Fomin: *Enumeration of plane curves and labeled floor diagrams*

Brugalle and Mikhalkin used tropical geometry to obtain a combinatorial description of Gromov-Witten invariants of projective spaces. We reformulate this description in the language of labeled floor diagrams, a particular class of acyclic oriented graphs with weighted edges and labeled vertices. We show that labeled floor diagrams of genus 0 are equinumerous to labeled trees, and therefore counted by the celebrated Cayley's formula. The corresponding bijections lead to interpretations of Kontsevich numbers (the genus-0 Gromov-Witten invariants of the projective plane) in terms of certain statistics on trees.

This is joint work with Grisha Mikhalkin.

Ezra Miller: *Applications of binomial commutative algebra*

The commutative algebra of ideals generated by binomials comes with inherent combinatorics. I will survey some past, present, and potential future applications of this combinatorics to hypergeometric systems, combinatorial game theory, and the dynamics of chemical reactions. Part of the emphasis here is on natural occurences of binomial ideals that are not prime.

Dan Laksov: *A formalism for equivariant cohomology*

About ten years ago Letterio Gatto suggested a way to interpret Schubert calculus in terms of exterior products. The resulting theory is so natural that it is astonishing it has not been observed before. It has many advantages to earlier descriptions of the cohomology of grassmannians:

- it gives a technically easy way to obtain the basic formulas of Schubert calculus;
- it is computationally very convenient
- it is general
- it suggest a generalization to limits of grassmannians
- by base change in the ring we take exterior powers of, we easily, and in a natural way, obtain equivariant cohomology, quantum cohomology, and equivariant quantum cohomology of grassmannians.

We shall explain a small detail in the theory, showing how the equivariant cohomology leads to a representation of symmetric polynomials.

Anders S. Buch: *Quantum K-theory of Grassmannians*

The Gromov-Witten invariants of a homogeneous space X give the number of rational curves of fixed degree that meet three general Schubert varieties, at least when this number is finite. When there are infinitely many such curves, then the moduli space of stable parametrizations of the curves is a projective variety. The K-theoretic Gromov-Witten invariants are the Euler characteristic of such varieties, and were used by Givental and Lee to define a quantum K-theory ring of X. I will present structure theorems for this ring when X is a Grassmann variety of type A, and a formula for the K-theoretic Gromov-Witten invariants that generalizes earlier work with Kresch and Tamvakis. This is joint work with L. Mihalcea.

William Fulton: *Character formulas*

In this expository talk, we show how a Riemann-Roch formalism leads to a simple proof of a general formula for restrictions of equivariant line bundles to fixed points. On homogeneous varieties it gives Weyl's character formula, and on toric varieties it gives Brion's formula for lattice points in polytopes. This is based on ideas of George Quart in the 1970's and recent conversations with Bill Graham.