

## **Prizes and Productivity: How Winning the Fields Medal Affects Scientific Output**

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**Abstract:** Knowledge generation is key to economic growth, and scientific prizes are designed to encourage it. But how does winning a prestigious prize affect future output? We compare the productivity of Fields medalists (winners of the top mathematics prize) to that of similarly brilliant contenders. The two groups have similar publication rates until the award year, after which the winners' productivity declines. The medalists begin to "play the field," studying unfamiliar topics at the expense of writing papers. It appears that tournaments can have large post-prize effects on the effort allocation of knowledge producers.

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## Prizes and Productivity: How Winning the Fields Medal Affects Scientific Output

George J. Borjas and Kirk B. Doran

I look forward to proving more theorems. I hope the weight of this prize doesn't slow me down.

—Stanislav Smirnov, Fields Medalist, 2010

### I. Introduction

The production of knowledge is central to long-term economic growth. Yet little is known about how knowledge is produced, making it difficult to predict which types of incentives are most effective in eliciting effort from knowledge producers. Prizes are a common incentive for knowledge production; hundreds of scientific prizes are awarded throughout the world and across all scientific disciplines. Although these prizes are frequently awarded with the explicit goal of inspiring more and better scientific work (Scotchmer, 2006), a question remains: are they effective?

An extensive theoretical and empirical literature examines how the presence of potential future rewards (whether a promotion to CEO or winning a golf tournament) elicits optimal efforts from the tournament participants in their effort to win the contest.<sup>1</sup> This literature emphasizes the incentive effects of the prize *prior* to the completion of the tournament. But what happens to the productivity of tournament winners *after* they win the prize?<sup>2</sup> Standard models of

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<sup>1</sup> Lazear and Rosen (1981) give the classic presentation of the tournament model; see also Rosen (1986). Empirical evidence on the productivity effects includes Ehrenberg and Bognanno (1990), Knoeber and Thurman (1994), and Main et al. (1993).

<sup>2</sup> Some recent studies address this question in the context of job promotions. Lazear (2004) offers an important discussion of the statistical problems introduced by mean reversion in the transitory component of productivity when measuring post-promotion productivity effects; see also the related empirical work in Anderson et al. (1999), and Barmby et al. (2012).

labor supply suggest that the post-prize impact of a big win could be significant, especially when the award is granted at a young age.

This paper examines the impact of winning the Fields Medal on the post-medal productivity and research choices of mathematicians.<sup>3</sup> The Fields Medal is the most prestigious award in all of mathematics, awarded every four years to mathematicians under the age of 40. Established by the Canadian mathematician John Charles Fields, the medal is often thought of as the “Nobel Prize of Mathematics.”<sup>4</sup> Inspired by the desire to promote mathematical cooperation and research around the world, Fields used his position as Chairman of the Organization Committee of the International Mathematical Congress to bring his idea to fruition. In a memo dated January 12, 1932, Fields described his vision:

It is proposed to found two gold medals to be awarded at successive International Mathematical Congresses for outstanding achievements in mathematics. Because of the multiplicity of the branches of mathematics and taking into account the fact that the interval between such congresses is four years it is felt that at least two medals should be available.

In the same document, Fields explained the motivation for the award: “while [the medal] was in recognition of work already done it was at the same time intended to be an encouragement for *further achievement* on the part of the recipients and a stimulus to renewed effort on the part of others” [emphasis added]. In other words, not only would the existence of the prize solicit effort from the participants in this elite tournament, it would also encourage additional (i.e., post-medal) effort on the part of the winners.

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<sup>3</sup> Zuckerman (1996) documents that the research output of Nobel Prize winners declines after winning the prize. Her descriptive evidence, however, is likely contaminated by the late age of the winners and the possibility of mean reversion because the comparison group is less productive prior to the awarding of the prize. Chan et al (2013) and Bricongne (2014) present related studies of the impact of the John Bates Clark Medal on the productivity of economists, and reach conflicting results. Finally, Azoulay et al. (2011) explore the impact of funding at the start of a scientist's career on subsequent productivity.

<sup>4</sup> Partly due to jealousy and conflict between Alfred Nobel and the Swedish mathematician Magnus Gotha Mittag-Leffler, Nobel famously left mathematics out of his list of recognized disciplines when he founded the prize that bears his name (Tropp, 1976). Ironically, Fields and Mittag-Leffler were close friends.

Using administrative data from the American Mathematical Society (AMS) and the Mathematics Genealogy Project (MGP), we examine the shape of the age-productivity profile of these exceptional mathematicians along a number of dimensions, including the number of papers published, citations received, and students mentored. Our empirical analysis exploits the fact that only a subset of the great mathematical contributions in the past 80 years resulted in Fields medals, and that this subset was partly determined by arbitrary factors such as the quadrennial timing of the award, the age restriction, and subject-level biases.

Our main set of results uses the set of winners from a broader set of prizes for great mathematical achievement (awards which are themselves good predictors of winning the Fields Medal) to construct a representative group of brilliant mathematicians who can be thought of as “contenders” for the medal. Our analysis compares the research output of the medalists with that of the losing contenders. The age-productivity profile of the two groups is similar until a particular mathematician wins the Fields Medal (or does not win it). Remarkably, the productivity of the Fields medalists *declines* noticeably relative to that of the contenders in the post-medal period, regardless of whether we measure productivity in terms of papers published, citations received, or students mentored.

The award of a Fields Medal enhances a mathematician’s professional reputation far more than any other potential event in the mathematics profession. Although the prize money is itself negligible, the medal can certainly lead to a substantial increase in “wealth” (both in terms of dollars and in intangible measures of academic prestige) if the mathematician chooses to leverage the award in the marketplace. The neoclassical labor-leisure model predicts that the expansion in the opportunity set (which essentially captures a broadly defined “wealth effect”) should induce the medalists to consume more leisure in the post-medal period. Moreover, the

wealth effect can also lead to a shift in the research strategy pursued by the Fields medalists: they are now free to “play the field” and pursue topics in different areas of mathematics (or even outside mathematics) that they may find interesting or worthwhile and have a high consumption value.

We employ the notion of “cognitive mobility” (Borjas and Doran, 2014) to capture the transition in the *space of ideas* as knowledge producers move from one research topic to another over the course of a career. The AMS data classifies each published paper into one of 73 specific and narrow mathematical fields. It turns out that there is a crucial link between a mathematician’s propensity for cognitive mobility across mathematical fields and the awarding of a Fields Medal. Specifically, while medalists and contenders have similar cognitive mobility rates initially, the medalists exhibit a far greater rate of mobility in the post-medal period. Because cognitive mobility is costly (e.g., additional time is required to prepare a paper in an unfamiliar field), the increased rate of cognitive mobility reduces the medalists’ rate of output in the post-medal period. The data suggest that about half of the decreased rate of output is due to the increased propensity for “trying out” unfamiliar fields, often outside pure mathematics.

Every four years, the greatest mathematicians in the world gather to select and honor new medalists and to remind them that the Fields Medal is meant to encourage their future achievement. In fact, the medal reduces the rate of publication and the likelihood that its winners produce great achievements in pure mathematics. In short, at least in this context, the net productivity impact of selecting winners on the basis of a tournament depends crucially on what happens as the winners adjust their behavior to take advantage of the expansion in the opportunity set.

## II. Historical Background

The first Fields Medals were awarded soon after Fields' 1932 memo. The medals are traditionally awarded during the opening ceremony of the quadrennial International Congress of Mathematicians (ICM). In 1936, the medals were awarded to two mathematicians. Because of World War II, the medals were not awarded again until 1950, when they were again given to two mathematicians. Since 1950, the Fields Medal has been awarded quadrennially, to two, three, or four mathematicians in each cycle.

The initial moneys available to fund the medals were the result of an accidental surplus of funds left over after the 1924 ICM. These funds, accompanied by the bequest of Fields himself, allowed for the granting of two medals. In 1966, an anonymous donor made additional funds available allowing four medals to be awarded in each of the next two cycles (ICM, 1966). As a result, the number of Fields Medals awarded in any given 4-year cycle was not mainly determined by how many mathematicians had made fundamental advances in the relevant time period. Instead, the number often depended on how much income had accumulated in the Fields Medal account, on the availability of private anonymous donors, and on an upper limit of (initially) two or (later) four medals to be awarded by any particular Congress (ICM, 2006).

As noted earlier, the Fields Medals were designed partly to promote future mathematical achievement on the part of the recipients. This goal is sufficiently important that it has been repeated verbatim and expounded upon at nearly every award ceremony. For example, in the 1954 Congress, eminent mathematician Hermann Weyl spoke movingly to the winners: "The mathematical community is proud of the work you both have done. It shows that the old gnarled tree of mathematics is still full of sap and life. Carry on as you began!"

From its inception, the committees have interpreted Fields' desire for future encouragement to mean that the medal should be awarded to mathematicians who are “young” (ICM, 1936), and the word “young” has consistently been interpreted to mean that the medal may only be awarded to mathematicians under the age of 40 (ICM, various issues).<sup>5</sup> Most recently, the 2006 committee explicitly stated its requirement that a mathematician qualifies for the Fields Medal only if he has not yet turned 40 as of January 1 of the year in which the Congress meets (ICM, 2006).

The institutional restrictions on the number and age distribution of the Fields medalists introduce arbitrary variation in which subset of great mathematicians of the past eighty years received the award and which did not. As a result, many mathematicians who are widely perceived as “great architects of twentieth-century mathematics,” even for work done at an early age, did not receive the Fields Medal (Tropp, 1976). There are numerous such examples. The American Mathematical Society said of mathematician George Lusztig: “[His work] has entirely reshaped representation theory and in the process changed much of mathematics” (AMS, 2008, p. 489). Although “Lusztig's exceptional mathematical ability became evident at an early stage of his career,” and “it can be no exaggeration to say that George Lusztig is one of the great mathematicians of our time,” he did not receive the Fields Medal (Carter, 2006, pp. 2, 42).

Similarly, the Norwegian Academy of Science and Letters (Solholm, 2010) cited John Tate for “his vast and lasting impact on the theory of numbers,” claiming that “many of the major lines of research in algebraic number theory and arithmetic geometry are only possible

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<sup>5</sup> The age restriction has been applied consistently over time. For example, the 1998 ICM stated: “As *all* the Committees before us, we agreed, . . . to follow the established tradition and to interpret the word ‘young’ as ‘at most forty in the year of the Congress’” [emphasis added].



because of [his] incisive contributions and illuminating insights.” Nevertheless, Tate also did not receive a Fields Medal.

In fact, considering the number of mathematicians who are regularly lauded by the various National Academies of Sciences and Mathematical Societies for (re)inventing new subfields of mathematics, it is clear that the 52 Fields Medals that have been awarded (as of 2013) are insufficient to cover even half of all the great achievements that have made modern mathematics possible. Hence it should not be particularly surprising that Robert Langlands, a mathematician whose work specifically inspired and made possible the contributions of at least two Fields Medalists (Laurent Lafforgue and Ngô Bảo Châu), and who founded the most influential program connecting number theory and representation theory, did not receive the Fields Medal himself.

Historians of the Fields Medal have also documented the “bias” that causes some fields and styles of mathematics to be better represented among winners (Monastyrsky, 2001). For example, Langlands (1985, p. 212) wrote of mathematician Harish-Chandra: “He was considered for the Fields Medal in 1958, but a forceful member of the selection committee in whose eyes Thom [one of the two Fields medalists that year] was a Bourbakist was determined not to have two. So Harish-Chandra, whom he also placed on the Bourbaki camp, was set aside.”<sup>6</sup>

Similarly, the arbitrary age cut-off and the four-year periodicity of the award work together to exclude mathematicians who obviously should have received the medal. The *New York Times* obituary of Oded Schramm states: “If Dr. Schramm had been born three weeks and a

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<sup>6</sup> The goal of the French Bourbaki group was to write down all of mathematics as a linear development from general axioms. The arbitrariness of the decision to exclude Harish-Chandra is doubly ironic: “Harish-Chandra would have been as astonished as we are to see himself lumped with Thom and accused of being tarred with the Bourbaki brush, but whether he would have been so amused is doubtful, for it had not been easy for him to maintain confidence in his own very different mathematical style in face of the overwhelming popular success of the French school in the 1950s” (Langlands, 1985, p. 212).

day later, he would almost certainly have been one of the winners of the Fields Medal...But the Fields Medals, which honor groundbreaking work by young mathematicians, are awarded only once every four years and only to mathematicians who are 40 or under. Dr. Schramm was born on Dec. 10, 1961; the cutoff birth date for the 2002 Fields was Jan. 1, 1962. Wendelin Werner, a younger mathematician who collaborated with Dr. Schramm on follow-up research, won a Fields in 2006” (Chang, 2008).

In short, while it is tempting to claim that the 52 Fields medalists are in a class by themselves, and that there are no losing contenders with equivalent or better early achievements, this view does not correspond with what mathematicians themselves have written. As the ICM noted: “we must bear in mind how clearly hindsight shows that past recipients of the Fields’ medal were only a selection from a much larger group of mathematicians whose impact on mathematics was at least as great as that of the chosen” (ICM, 1994). The arbitrariness in the number, timing, and field distribution of Fields medalists means that a similarly great group of “contenders” should exist that can be contrasted with the winners in a difference-in-differences strategy to determine how winning the medal influences productivity and research choices.

### **III. Data**

To measure the life cycle productivity of elite mathematicians, we use the comprehensive data contained in the AMS *MathSciNet* archives. The AMS provided us with a database that reports the number of papers published by every mathematician in the world, by field and year, since 1939. The AMS professional staff assigns each publication in mathematics to one of the many fields that make up the discipline (and this information will prove useful below). Our database contains the author-year-field information at the two-digit field level, classifying every publication over the 1939-2011 period into one of 73 different fields. The database also contains

information on the number of citations received by the papers. It is important to note, however, that the AMS citation data is incomplete. In particular, it only counts citations in a limited number of journals (which include the most important journals in mathematics), and only reports the post-2000 citations received by a paper (regardless of when the paper was published).

We wish to determine what the post-medal career path of Fields medalists would have looked like had they not been awarded the medal. Because of the capricious events affecting the selection of the subset of great mathematicians who received the medal, we conjecture that there should exist a comparison group of mathematicians who did similarly path-breaking work before the age of 40, but who did not receive the medal and can serve as a control group.<sup>7</sup>

We use a systematic and easily replicable method for constructing the set of “contenders.”<sup>8</sup> Specifically, our construction of the control group starts out by including the winners of six other major mathematical awards with roughly similar goals as the Fields Medal. It turns out that winning any one of these prizes is a good predictor for receiving a Fields Medal.

First, we consider the two most prestigious general mathematics prizes (after the Fields Medal), which tend to be given closer to the end of a mathematician’s career. Both the Abel Prize and the Wolf Prize cover the entire breadth of the mathematics discipline and are only given to mathematicians who have made extraordinary contributions. The Abel Prize, which has

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<sup>7</sup> An alternative to the difference-in-differences approach would be the use of instrumental variables. The obvious choice of an instrument for winning the Fields Medal is given by the combination of the quadrennial timing of the award and the age cut-off: some mathematicians have almost four more years to compete for the Fields Medal than others. While the resulting variation in the maximum number of “eligible work years” is positively related with winning the Fields Medal in the sample of contenders constructed below, the Angrist-Pischke multivariate  $F$ -tests of excluded instruments show that the relationship is not sufficiently strong to make it a useful instrument by itself.

<sup>8</sup> We could also rely on statistical matching based on papers per year or citations per paper to construct a sample of contenders from the universe of all mathematicians. This more mechanical approach, however, ignores the valuable information embedded in the profession’s willingness to publicly acclaim a particular person’s contributions with a scarce award. It would also be difficult to operationalize because a key variable in the matching algorithm would be year of birth (which determines eligibility for the Fields Medal). The birth year would need to be uncovered one at a time through archival research or one-on-one contact for the universe of potential matches in the AMS data.

a significantly higher monetary value than the Fields Medal (nearly \$1 million versus \$15,000), began to be awarded in 2003 to one or two mathematicians a year.<sup>9</sup> The Wolf Prize has been awarded annually since 1978, typically to two mathematicians (although no prize has been awarded in some years). Any mathematician who won either of these prestigious awards (and did not win the Fields Medal) is clearly a key formulator of modern mathematics and automatically becomes part of our group of potential contenders.

In addition to these two general prizes, there are a number of prestigious area-specific prizes in mathematics. Specifically, we consider the four most prestigious area-specific awards for: algebra (the Cole Prize of the AMS); analysis (the Bôcher Prize of the AMS); geometry (the Veblen Prize of the AMS), and the study of Fourier series (the Salem Prize). We add into our group of potential contenders any mathematicians who won one of these four area prizes before the age of 40 (and did not win the Fields Medal).<sup>10</sup>

This algorithm yields the names of 92 potential contenders who contributed significantly to at least one of the key subject areas of mathematics or to mathematics as a whole, but who did not receive the Fields Medal. There is a very strong correlation between winning any of these prizes and winning the Fields Medal: 52 percent of the Fields medalists also won at least one of these prestigious awards. The predictive power of each prize is as follows: 5 out of the 13 Abel Prize winners also won the Fields; as did 13 out of the 54 Wolf Prize winners; 3 out of the 26

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<sup>9</sup> The Abel Prize creates a multi-stage tournament for mathematicians. As in Rosen (1986), Fields medalists may wish to keep participating in the tournament in order to receive the sizable monetary award associated with the Abel Prize. The changed incentives, however, are unlikely to affect our results because the Abel Prize began late in the sample period.

<sup>10</sup> There are many other mathematical prizes around the world, but they are less prestigious or worse predictors of winning the Fields Medal. For example, none of the winners of the AMS Cole Prize for Number Theory went on to win the Fields Medal.

Cole Prize winners; 4 out of the 32 Bôcher Prize winners; 4 out of the 29 Veblen Prize winners; and 7 out of the 48 Salem Prize winners.

Our empirical strategy requires us to determine if a mathematician is eligible for the Fields Medal in any particular cycle, so that we need to observe the mathematician's date of birth. Although the AMS data does not provide this information, we ascertained the birth date (through internet searches for each mathematician's *curriculum vitae* or personal contact) for all Fields medalists and for almost all of the potential contenders. The systematic archival of publications by *MathSciNet* started in 1939, and some mathematicians in our sample published in their teenage years, so we restrict the study to those born in or after 1920. Further, we exclude the 6 potential contenders for whom we could not confirm a date of birth. This leaves us with a sample of 47 medalists and 86 potential contenders. The Appendix Table presents the combined list of all winners, a list that includes all the mathematicians mentioned in our historical survey.

There is obviously a great deal of variation in the mathematical significance and timing of the work of the potential contenders. For example, the narrowness of the area prizes suggests that the contribution of some of these winners, although very important in that particular area, may not have the "breadth" required to generate sufficient interest in the broader community of mathematicians. Similarly, some of the contenders (who perhaps went on to win one of the general prizes) may have produced their best work after their eligibility for the Fields Medal ended. Hence we whittle down the list of 86 potential contenders by examining how often other mathematicians cite the work that the contenders produced during the years they were eligible for the Fields Medal.

A recipient of the Fields Medal cannot have turned 40 after January 1 of the year in which the medal is awarded. For example, the 2010 medal cycle would have been the last cycle

for a person born anytime between January 1, 1970 and December 31, 1973. Even though the contenders born in this time frame did not win in their last shot at the medal, the incentives for “impressing” the Fields Medal committee ended in 2010. Hence we assign the year 2010 as the “medal year” for these contenders to separate the pre- and post-medal periods. We used a similar exercise to ascertain the medal year of all the contenders in our sample. A mathematician’s eligibility period is then given by the years between the mathematician’s first publication and the medal year.

We calculate the annual rate of citations generated by a potential contender during his eligibility period by dividing the total number of citations received by papers published in this period (cumulative as of October 2011) by the number of years in the eligibility period. We then define the final set of contenders as the 43 mathematicians in this group whose annual eligibility-period citation rate is above the median. In other words, our final group of contenders represents “la crème de la crème” of mathematicians who did widely recognized work during the eligibility period and who did not win the Fields Medal.<sup>11</sup>

Table 1 reports summary statistics for the sample of Fields medalists as well as for the control group (using both the final list of contenders with above-median citations, as well as the group of all 86 prize winners).<sup>12</sup> The table also reports comparable statistics (when available) for a sample of “professional mathematicians,” which we define as the group of mathematicians in the AMS archive whose first and last published papers span at least a 20-year period.

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<sup>11</sup> An alternative way of defining the final set of contenders would be to calculate the annual rate of papers published during the eligibility period, and select the 43 mathematicians whose annual rate of output is above the median. The evidence reported below is similar if we used this alternative definition. We also accounted for the fact that the nature of the AMS citation data could imply that the number of citations received by more recent mathematicians may be greater than the number received by mathematicians active in the 1950s and 1960s. We defined the final set of contenders based on the above-median ranking of the residual from a regression of the per-year number of citations in the eligibility period on the calendar year of first publication, and the results are very similar to those reported in the next two sections.

<sup>12</sup> We ignore the posthumous publications of the Fields medalists and contenders.

Obviously, both the Fields medalists and the contenders publish much more and receive many more citations than the average mathematician. There is, however, relatively little difference in measured productivity between the final group of contenders and the Fields medalists. The medalists published 3.1 papers per year during their career, as compared to 3.6 papers for the contenders. The typical paper published by a medalist received 21.0 citations, as compared to 17.5 citations for the contenders. The average mathematician in both groups was born around 1950, and they each published their first paper at the early age of 23 or 24.

The table also summarizes the rate of output by age, calculating the average number of papers published annually by the medalists and the contenders between the ages of 20-39 and 40-59. The data reveal suggestive differences. The medalists and the contenders published essentially the same number of papers per year in the early part of the career (3.4 papers), but the medalists published 1.2 fewer papers per year after age 40. This striking pattern presages the nature of the empirical evidence that will be documented in subsequent sections.

#### **IV. The Fields Medal and the Age-Productivity Profile**

As noted earlier, despite the plethora of important prizes that a brilliant mathematician can potentially receive, the prestige of the Fields Medal is substantially greater than that of any other prize. In fact, the ICM Fields Medal announcement emphasizes that the prestige effect is far greater than the accompanying monetary award: “The Fields Medals carry the highest prestige of all awards in mathematics. This prestige does not derive from the value of the cash award, but from the superb mathematical qualities of the previous Fields Medal awardees” (ICM, 2010).

Nevertheless, it is obvious that the financial impact of the Fields Medal on a mathematician’s lifetime wealth is not limited to the \$15,000 monetary prize. Fields medalists

are likely to see a substantial expansion in their opportunity set, in terms of high-quality job offers, additional research funding, and many other career opportunities. It is conceivable, therefore, that the wealth effect (which includes money as well as the additional opportunities allowed by the substantial increase in professional prestige) could be sizable and could alter the medalist's post-medal behavior.

The neoclassical labor-leisure model suggests that the wealth effect should increase the consumption of leisure by the Fields medalists relative to that of the contenders. As a result, we should not be surprised if the "weight of the prize" does indeed slow the Fields medalists down. Moreover, the wealth effect might influence the mathematician's choice of research topics, either because the mathematician can now afford to explore topics that are essentially "consumption goods" or because the medalist feels that he can pursue "riskier" topics. These shifts in research interests, discussed in more detail below, also have productivity consequences.

We initially measure the productivity of the elite mathematicians by the number of papers published in each year. Figure 1 illustrates the life cycle trend in the average number of papers published by both the medalists and the control group (i.e., the subset of contenders who have above-median citations in the eligibility period). Specifically, the figure plots the average number of papers published per year by the medalists and the contenders at the prime of their career, *relative* to the medal year. The Fields medalists are plotted relative to the year they actually received the medal (a zero on the  $x$ -axis represents the year of the prize); the contenders are plotted relative to the last year of their eligibility for the medal (a zero on the  $x$ -axis represents the last year of their eligibility).

It is evident that the medalists and the contenders had very similar age-productivity profiles during the eligibility period, publishing around 3 to 4 papers per year. The figure also



shows, however, a dramatic drop in the annual rate of output for the medalists that coincides with their receipt of the Fields Medal. A decade or two after the Fields medal, the average medalist published around 1.5 fewer papers per year than the average contender.

Of course, these differences could be due to factors that cannot be controlled for by the graphical analysis, including individual fixed effects, calendar-year effects, and age differences. We stack the annual data in our panel of medalists and contenders, and estimate the regression model:

$$(1) \quad y_{it} = \delta_i + \delta_t + \alpha T_i + \beta(T_i \times F_i) + Z_i \gamma + \varepsilon,$$

where  $y_{it}$  gives the number of papers published by mathematician  $i$  in calendar year  $t$ ;  $\delta_i$  and  $\delta_t$  are vectors of individual and calendar-year fixed effects, respectively;  $T$  is a dummy variable indicating if the observation refers to the post-medal period;  $F$  is a dummy variable indicating if mathematician  $i$  won the Fields medal; and  $Z$  is a set of background characteristics that includes the mathematician's age (introduced as a fourth-order polynomial). The data panel contains one observation for each mathematician for each year between the year of the first publication and the most recent year of "potential activity" (if alive) or the year of death. The coefficient  $\alpha$  measures the difference in the annual rate of publication between the post- and pre-medal periods for the contenders, while the coefficient  $\beta$  measures the relative change in this gap for the Fields medalists.

Table 2 reports the estimated coefficients using a variety of alternative specifications. The first two columns report coefficients when the control group is formed by the sample of contenders with above-median citations in the eligibility period. Row 1 reports the simplest

regression model. The estimate of  $\alpha$  is small, suggesting no substantial difference in the average annual product of the contenders in the pre- and post-medal periods (after controlling for age and period effects). The estimate of  $\beta$  is negative and around -1.4, indicating a (relative) drop of more than one paper per year in the post-medal period for the Fields medalists. In other words, even after controlling for individual-specific productivity differences and aging effects, there is a sharp decline in the productivity of the medalists after they were awarded the medal.

The specifications changes reported in the remaining rows of Table 1 corroborate this finding. Row 2 uses the log number of papers per year as the dependent variable, but excludes from the regression those (relatively few) years where the elite mathematicians did not publish at all. The log papers regression shows a 24 percent decline in productivity in the post-medal period. Row 3 reports the coefficient from a quantile regression where the dependent variable is the median number of papers per year (using bootstrapped standard errors clustered at the mathematician level). The regression shows a decline of -0.7 papers per year in the post-medal period. Finally, the last two rows of the table use alternative methods for ascertaining the “medal year” in the sample of contenders: either at age 36 (the median age at which Fields medalists actually receive their medal) or age 40 (the maximum age of eligibility). The estimate of the coefficient  $\beta$  is robust to these alternative definitions.

The last two columns of the table report the estimated coefficients when the control group includes all 86 members of the sample of contenders (i.e., all the prize winners without any quality cutoff). Regardless of the method used to define the sample of contenders, the regression coefficients are similar. The data reveals that the Fields medalists produce between 0.9 and 1.4 fewer papers per year in the post-medal period (or roughly a 20 percent decline in productivity)

than would be predicted either from their previous output or from the output of other great mathematicians who did not win the highly coveted prize.<sup>13</sup>

The observed decline in the annual number of publications cannot be attributed to either the effect of mechanical mean reversion or to an “expectation bias” among the members of the award committee. Suppose that a mathematician’s observed productivity at a point in time has a transitory component. As Lazear (2004) notes, the population of mathematicians whose productivity is above some bar at time  $t$  will always show an average decline in productivity after time  $t$  due to mean reversion in the transitory component of output. To avoid this type of contamination, we restricted our analysis to contenders who had reached similarly high productivity levels in the Fields Medal eligibility period. As a result, any mean reversion should operate equally on both groups.

Second, the award committee members, containing some of the best mathematicians in the world, surely observe future predictors of productivity among the contenders that we cannot measure in a publication database, and they may be swayed by this private information in their discussions. Given the future expectations bias, therefore, it would not be surprising if the tournament winners do *better* in the post-tournament period. This bias would imply that our regression coefficients *understate* the true post-medal productivity decline.

We suspect, however, that this bias is less likely to be important in the context of the Fields Medal than in comparable tournaments in “softer” sciences, such as the John Bates Clark Medal in economics. First, publication lags in pure mathematics are considerably shorter than they are in economics; a mathematics paper may be in print six months after the proof is

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<sup>13</sup> We also examined the robustness of our results to the use of the Abel and Wolf prizes in the construction of the sample of contenders in two alternative ways: by excluding from the sample of contenders those mathematicians who won the Abel or Wolf prizes (yielding a sample of 35 contenders) or by *only* including in the sample of contenders those mathematicians who did win the Abel or Wolf prizes (8 contenders). The interaction coefficient  $\beta$  is -0.908 (0.458) in the first case and -2.553 (2.015) in the second.

complete. Second, mathematicians are, by necessity, far less open about discussing the nature of the theorems they are trying to prove before a paper has been peer-reviewed and accepted by a journal. Finally, even the private information that a mathematician is out to prove Hilbert's Eighth Problem is unlikely to influence the Fields Medal committee, as the resulting proof will either be correct or not and the committee cannot determine the validity of the proof in advance. In contrast, an interesting and fertile research agenda in applied economics can often produce exciting papers regardless of the direction in which the data points.<sup>14</sup>

The AMS data also allows us to examine other output effects of the Fields Medal. Table 3 re-estimates the basic regression model using alternative dependent variables. The dependent variable in row 1 is the probability that a mathematician publishes at least one paper in a given year. The relative probability of publishing a paper falls by about 11.8 percentage points for the medalists in the post-medal period. Row 2 uses the number of citations generated by papers written in year  $t$  as the dependent variable. Although the AMS data only reports the post-2000 citations for a paper regardless of when the paper was published, the calendar year fixed effects included in the regression model should control for the variation in citations between older and newer papers. The coefficient  $\beta$  is again negative and significant, suggesting a decline of about 44 citations annually for papers produced in the post-medal period.

Part of the decline in citations is attributable to the fact that the medalists are less likely to publish (and publish fewer papers when they do publish). Row 3 uses the number of citations per paper published in a given year as the dependent variable (excluding years when the mathematician did not publish at all). The material published by the medalists in the post-medal

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<sup>14</sup> The favorable productivity effect of the John Bates Clark medal reported in Chan et al. (2013) could be interpreted in this context. More recent work by Bricongne (2014), however, reports conflicting evidence, concluding that the Clark medal may perhaps even have a slightly adverse productivity impact. It seems, therefore, that no consensus yet exists on the sign of the Clark medal's effects.

period is, on average, less citation-worthy than the material published by the contenders. Even if the typical post-medal paper written by a medalist generates fewer citations, the medalists may be just as likely to hit a “home run.” We calculated a vintage-specific citation cutoff for papers published each year using the universe of publications in the AMS database. By definition, a mathematician hits a “home run” if the number of citations per paper published that year was above the 99.5<sup>th</sup> percentile for all mathematicians in the AMS database. Row 4 shows that the (relative) probability that a medalist hits a home run in the post-medal period declines by 15.6 percentage points. At the other extreme, a mathematician may “strike out” and write papers that are *never* cited. Row 5 shows that the medalist’s (relative) probability of striking out rises by 5.3 percentage points in the post-medal period.

Finally, many of these elite mathematicians devote considerable time and effort to training the next generation of mathematicians. In fact, biographies and laudations of their achievements emphasize the training and mentoring of students as evidence of their long-lasting impact on mathematics. We therefore also examine the impact of the Fields Medal on the medalists’ mentoring activities.

We obtained access to the data in the Mathematics Genealogy Project (MGP), and we merged the genealogy data with the AMS publication data. The MGP data identifies the intellectual progeny of the renowned mathematicians in our sample, as well as the year in which those students received their doctoral degree. We were able to match 104 of the 133 mathematicians using an MGP-AMS match provided by the administrators of the MGP. For 28 of the remaining 29 unmatched mathematicians, we were able to obtain information about the graduation years and names of their students from name-based searches of their curriculum vitae, obituaries, or unmatched online MGP entries. The merged data also provides information on the

research output of the students in their post-doctoral career.<sup>15</sup> The merged data, therefore, allows us to examine not only the impact of the Fields Medal on the number of students produced in the post-medal period, but also the impact on the quality of the students.

We are interested in the relation between the timing of the year in which a student becomes an elite mathematician's mentee and the year in which the mathematician receives (or does not receive) the Fields Medal. The mentoring agreement typically occurs two to four years before the student obtains his or her doctoral degree. We lag the MGP degree date by three years to approximate the year in which the mentoring relationship began.<sup>16</sup>

It turns out that Fields medalists are not only publishing fewer papers in the post-medal period, and that those papers are relatively less important, but they are also accepting fewer mentees under their wing. Row 6 of Table 3 reports the relevant coefficients when we estimate the regression model using the number of mentees as the dependent variable. The regression shows a (marginally significant) relative decline in the number of mentees accepted by the Fields medalists of about 0.1 students per year. The last two rows estimate the regression model using the mentee's total number of publications and citations over their career to date as dependent variables. The results show a pronounced decline in the quality-adjusted student output of Fields medalists in the post-medal period.

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<sup>15</sup> For students whose AMS identification numbers are listed in the MGP database, we use the AMS data to calculate their career papers and citations. Many mathematics doctorates, however, do not publish a single paper in their career (and the mode for those who do is a single publication with zero citations; see Borjas and Doran, 2012). The absence of a publication implies that the student will never appear in the *MathSciNet* database. We assume that the students who do not have an AMS identification number have zero lifetime publications and citations.

<sup>16</sup> The results are similar as long as we lag the degree date by 2 or more years.

### **An Alternative Control Group**

It is important to establish that the evidence reported in Tables 2 and 3 cannot be attributed to idiosyncratic properties of the method we used to construct the control group of contenders. In fact, it is easy to show that the results are almost identical if we pursued a drastically different method for constructing the sample of contenders. In addition to the various prizes discussed above, the mathematics profession honors a select number of elite mathematicians with invitations to be “plenary speakers” at the quadrennial International Congress of Mathematicians. The qualifications required for receiving such an invitation are clearly stated in the memo detailing the program committee’s responsibilities (ICM, 2007):

Plenary lectures are invited one-hour lectures to be held without other parallel activities... The lectures should be broad surveys of recent major developments, aimed at the entire mathematical community. Plenary speakers should be outstanding mathematicians and good lecturers. It is the privilege of the PC [Program Committee] to select the plenary speakers.

The program committees in charge of organizing the plenary lectures have typically scheduled around 20 plenary lectures at each ICM in recent decades. There have been 269 plenary speakers since the 1950 ICM, and these speakers include 34 of the 50 mathematicians awarded the Fields medals during this period. We again obtained the lifetime publication record for each of the invited plenary speakers from the AMS archives, and were able to determine the year of birth for all but eight of the speakers. We then re-estimated the basic regression model in equation (1) using the group of plenary speakers (who did not win the Fields medal) as the control group.<sup>17</sup>

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<sup>17</sup> It is interesting to note that 39 of the 86 contenders produced by the method based on receipt of mathematical prizes have given a plenary lecture, as have 21 of the 43 “final” contenders with an above-median citation rate in the eligibility period.

Table 4 summarizes the key results from our replication of the analysis. We illustrate the robustness of the evidence by estimating alternative specifications of the regression model, including using the group of all plenary speakers as contenders or only the group of plenary speakers who received an invitation to present a lecture by age 44.<sup>18</sup> In addition, the table illustrates the robustness of the results when these samples are further restricted to only include mathematicians whose citation rate during the eligibility period placed them in the top half of the group. Regardless of the exact definition of the control group, the evidence summarized in Table 4 is similar to that discussed earlier (both qualitatively and quantitatively). Fields medalists publish around one fewer paper annually after they receive the medal; the papers they do publish get cited less often; and they have a lower probability of hitting a home run and a higher probability of hitting a strikeout. Because the evidence is robust to the definition of the control group, the analysis presented in the remainder of this paper reverts to the simpler definition of a control group that uses the sample of mathematicians who have received one of the six prestigious awards noted earlier.

## V. Cognitive Mobility

The post-medal productivity of Fields medalists, in terms of the number of papers published, citations generated, or students mentored, is lower than would have been expected. A question immediately arises: what exactly are the medalists doing with their time in the post-medal period?

One obvious possibility is that the expansion in a mathematician's opportunity set implied by the Fields Medal introduces incentives to consume more leisure—along the lines of

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<sup>18</sup> By restricting the sample to plenary speakers who gave their lecture before age 44, we are indirectly attempting to construct a sample of outstanding mathematicians that includes those who “just missed” their chance for a Fields medal because of the interaction between the age restriction and the quadrennial timing of the award.



the wealth effect in the neoclassical labor-leisure model. As long as leisure is a normal good, the increase in the opportunity set associated with the Fields medal (which includes the value of the additional prestige, concurrent invitations, job offers, grant opportunities, etc.) could lead to the medalists behaving in the predicted fashion and increasing their consumption of leisure. The increased leisure leaves less time for writing papers and supervising students.

In fact, the neoclassical labor-leisure model has a second implication: the Fields Medal should increase his consumption of *all* normal goods. A medalist could respond by increasing his consumption of “enjoyable research” in fields outside of pure mathematics, and perhaps begin to dabble in such disciplines as biology and economics.<sup>19</sup> Moreover, the medalist may now perceive a freedom to pursue research topics that lead to riskier outcomes than he would have pursued otherwise.<sup>20</sup> These shifts in research interests may affect productivity.

We apply the concept of “cognitive mobility” to analyze the choice of post-medal research topics by the elite mathematicians in our sample. As noted in Borjas and Doran (2014), knowledge producers who are conducting research on a particular set of questions may respond to changed opportunities by shifting their time, effort, and other resources to a different set of questions. Cognitive mobility then measures the transition from one location to another in idea space.

We compare the cognitive mobility rates of the medalists and contenders in the post-medal period. As noted earlier, the AMS data provides information not only on the annual output (as measured by papers and citations) of mathematicians, but also categorizes each paper into

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<sup>19</sup> Levin and Stephan (1991) and Stern (2004) suggest that some scientists, particularly theoretical ones, derive consumption value from doing research they enjoy.

<sup>20</sup> Since Arrow (1965) and Stiglitz (1969), it is well known that a wealth increase may prompt a utility-maximizing agent to undertake riskier investments. Levhari and Weiss (1974) extended this insight to the human capital framework.

one of the 73 fields that make up the discipline of mathematics and related subjects. Because of the large number of fields, it is obvious that we need to reduce the dimensionality of the space of ideas in order to operationalize the concept of cognitive mobility in the current context.

Assume that a mathematician's career begins the year he publishes his first paper. We can then examine the distribution of a mathematician's research topics in, say, the first  $x$  years of his career. The AMS data allows us to determine the *modal* field of the papers published in those years, as the mathematician was getting his career started and signaling his "quality" to the rest of the profession.

Although mathematics is composed of 73 fields, some of these fields are intellectually close to the modal field, while others are unrelated. The notion of cognitive mobility, therefore, should incorporate the fact that a move between the modal field and any other field may be "cognitively close" or "cognitively far." To determine the cognitive distance between any two fields, we calculated a matrix with elements  $[f_{ij}]$  showing the fraction of references made by papers published in field  $i$  to papers published in field  $j$ .<sup>21</sup> To illustrate, suppose that the modal field was Partial Differential Equations. The three most closely related fields (with the three largest values of  $f_{ij}$ ) are Partial Differential Equations itself, Global Analysis, and Fluid Mechanics. These three fields account for 72 percent of all references made by papers published in Partial Differential Equations. At the other extreme, papers published in Partial Differential Equations *never* referenced papers published in either General Algebraic Systems or **K**-Theory.

It turns out that we typically do not need to expand the definition of "cognitively close" beyond 15 fields to capture almost all the references made by papers published in field  $i$ . For

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<sup>21</sup> It is not possible to estimate this matrix with the data that the AMS provided us. We instead purchased citation data from the ISI Web of Science to calculate these distance measures (see the Data Appendix to Borjas and Doran, 2012, for details). The calculation of the matrix uses all publications in the 1979-2009 period.

example, 77.6 percent of all references made in Partial Differential Equations are to the top 5 fields, 87.4 percent are to the top 10, and 92.5 percent are to the top 15. This clustering around a very small number of fields is quite representative of the discipline of mathematics. In particular, 93.1 percent of the references in papers published in the median field of mathematics are made to papers published in only 15 other fields (the respective statistics for the 10<sup>th</sup> and 90<sup>th</sup> percentile fields are 87.6 and 97.5 percent).

Of course, it is not uncommon for elite mathematicians to move within a small (and often related) set of fields in the early part of their career. To capture this oscillation, we expand the definition of the “modal field” to include either the most common *or* the second most common field in the early part of a mathematician’s career. For each of these two modes, we then constructed the set of the 15 most related fields. Our cognitive mobility variable then indicates if the mathematician moved outside the two modal fields *and* all related fields (in other words, if the mathematician moved out of the potential maximum of 30 fields that broadly define his initial research interests).

To easily illustrate the trends in cognitive mobility, we first define the “early career period” as the eligibility period for the Fields Medal (i.e., the years before the Fields medalist won the medal or the years in which contenders were eligible for the medal). We then calculate the probability that papers published in each year of a mathematician’s career are in a different field than the two modal (and related) fields. Figure 2 plots this measure of cognitive mobility. As before, Fields medalists are plotted relative to the year they actually received the medal, and the contenders are plotted relative to the last year of eligibility.

The probability that either the medalists or contenders strayed from their “comfort zone” prior to the medal year is small, around 5 percent a year for mathematicians in either group. This

similarity, however, breaks down dramatically in the post-medal period. The rate of cognitive mobility doubled to 10 percent for the contenders, but rose dramatically for the medalists, quintupling to 25 percent. In short, the data reveal that the awarding of the Fields Medal is associated with a strong increase in the likelihood that a mathematician tries out fields that are very distant from those fields that established his reputation.<sup>22</sup>

To determine if this correlation persists after controlling for individual and period fixed effects, we use the AMS data to construct a panel where an observation represents a paper published by each mathematician. In particular, let  $p_{int}$  be an indicator variable set to unity if the field of the  $n^{\text{th}}$  paper published by mathematician  $i$  (and published in year  $t$ ) differs from that of the modal and related papers in the baseline period. We then estimate the regression model:

$$(2) \quad p_{int} = \delta_i + \delta_t + \alpha T_i + \beta(T_i \times F_i) + Z_i \gamma + \varepsilon.$$

Table 5 reports the relevant coefficients ( $\alpha$ ,  $\beta$ ) using several alternative specifications. As with the illustration in Figure 2, the first row of the table uses the publications in the eligibility period to define the set of fields that make up the mathematician's comfort zone. We illustrate the robustness of our results by using either the two modal fields (and up to 30 related fields), or just simply *the* modal field (and its 15 related fields). Regardless of the specification, the awarding of a Fields medal substantially increases the rate of cognitive mobility. Even after controlling for individual-specific fixed effects, the awarding of the Fields Medal increases the probability of a move by between 16 and 21 percentage points.

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<sup>22</sup> It is worth noting that the trends in cognitive mobility are inconsistent with a mean reversion interpretation of the post-medal productivity of the Fields medalists.

The next two rows of the top panel conduct sensitivity tests by using alternative definitions of the “early career” period used to construct the mathematician’s comfort zone. Row 2 uses the first 3 years of the career, while row 3 uses the first 5 years. Similarly, the regressions in Panel B use the entire sample of contenders (without any quality cutoff) to estimate the model. Regardless of the specification, the awarding of the Fields Medal has a positive and significant impact on the probability that a mathematician engages in cognitive mobility.<sup>23</sup>

The freedom to try out new things, however, does not come cheap. Cognitive mobility, like any other type of move, can be costly. The mathematician is exiting a field where he has remarkable technical skills and attempting to prove theorems in areas where his intuition may not be as strong and where the proofs may require a new set of tools. It would not be surprising, therefore, if it takes longer to produce a paper after the mathematician has engaged in cognitive mobility.

Define the duration of a “preparation spell” as the length of time elapsed (in years) between any two consecutive papers in a mathematician’s career. We estimated a regression model to measure the relation between the length of the preparation spell for paper  $n$  ( $n > 1$ ) and cognitive mobility:

$$(3) \quad \pi_{int} = \delta_i + \delta_t + \lambda_F(F_i \times p_{int}) + \lambda_C(C_i \times p_{int}) + Z_i \gamma + \varepsilon,$$

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<sup>23</sup> We also examined the probability that a mathematician conducts research outside pure mathematics. Specifically, we constructed a variable indicating if a paper was in any of the following “applied” areas: “History and biography,” “Statistics,” “Computer science,” “Geophysics,” “Game theory, economics, social and behavioral sciences,” “Biology and other natural sciences,” “Information and communication, circuits,” or “Mathematics education.” The estimated regression coefficient suggests that the (relative) probability of a Fields medalist publishing in one of these applied areas in the post-medal period rose by 75 percent.

where  $\pi_{int}$  gives the length of the preparation spell required to write paper  $n$ ;  $p_{int}$  is the indicator variable set to unity if paper  $n$  involved a cognitive move, and  $C_i$  is a variable indicating if mathematician  $i$  is a contender ( $C_i = 1 - F_i$ ). Table 6 summarizes the estimates of the vector  $(\lambda_F, \lambda_C)$  using alternative specifications of the model. It is evident that cognitive mobility is associated with a longer preparation spell for both the medalists and the contenders, and the effect is numerically important. A cognitive move increases the length of the preparation spell by between 0.16 and 0.23 years.

In sum, the data indicate that the Fields medalists engaged in more cognitive mobility in the post-medal period *and* that cognitive mobility imposes a cost; it takes longer to produce a paper. This behavior, therefore, will inevitably result in a reduced rate of publication for the medalists in the post-medal period.

The regression coefficients can be used to conduct a back-of-the-envelope calculation that determines how much of the observed decline in productivity was due to cognitive mobility. The results in Table 5 indicate that the awarding of the Fields Medal increased the probability of cognitive mobility for a paper published in the post-medal period by around 15 percent. Both the medalists and the contenders published 4 papers per year at the time the medal was awarded (see Figure 1). Using this rate of output as the baseline, the regression coefficient in Table 5 indicates that the awarding of the medal led to a 0.6 increase (or  $0.15 \times 4$ ) in the number of papers published annually in an unfamiliar field.

At the same time, Table 6 shows that cognitive mobility increases the length of a preparation spell by about 0.2 years. Putting these results together implies that the increased incentive for cognitive mobility in the post-medal period and the longer preparation spell reduces the amount of “effective” time available in a given year by about 0.12 years (or the 0.6 papers

published in an unfamiliar field times the 0.2 longer years it takes to produce such a paper). In rough terms, therefore, we expect a 12 percent decline in the number of papers that a medalist published annually in the post-medal period simply because cognitive mobility diverts 12 percent of his time to other uses (e.g., learning new skills). As we saw in Table 2, there was a 24 percent decline in annual output. The increased experimentation exhibited by Fields medalists in the post-medal period can account for about half of the decline in productivity.

It is important to emphasize that the decline in productivity resulting from the wealth effect that increases leisure is conceptually different from the decline induced by the increased experimentation. Although the cognitive mover publishes fewer papers, those papers may provide a social benefit. The medalist is applying his talents to unfamiliar questions, and may generate important insights in areas that were previously under-served by exceptional mathematical talent.

In fact, among the great architects of late twentieth century mathematics in our sample, there are three well-known examples in which a Fields medalist who made extraordinary contributions to a specific area of pure mathematics went on to mathematize a distant applied subject later in their career. René Thom (and his development of singularity/catastrophe theory), David Mumford (and the mathematics of vision and pattern theory), and Stephen Smale.

Smale's experience is particularly illuminating. Gleick (1987, p. 45) recounts what happened soon after Smale proved a pure mathematics result (the Generalized Poincaré conjecture) that helped earn him the Fields Medal.

Smale [was]. . . already famous for unraveling the most esoteric problems of many-dimensional topology. A young physicist, making small talk, asked what Smale was working on. The answer stunned him: "Oscillators." It was absurd. Oscillators—pendulums, springs, or electrical circuits—were the sort of problem that a physicist finished off early in his training. They were easy. Why would a great mathematician be studying elementary physics?

Even if a young physicist considered Smale's new choice of topic simplistic and absurd, the enormity of the mathematician's previous achievements insulated him from any real loss of prestige. Smale's post-medal experimentation built the mathematical foundation of chaos theory. In fact, Smale went on to make important contributions in biology, astronomy, and even in theoretical economics.

## **VI. Summary**

A vast literature explores the impact of tournaments, contests, and prizes on the productivity of tournament participants, analyzing the implications of pre-award productivity effects for the efficient design of incentive mechanisms. A working assumption in this literature is that the labor supply consequences of actually winning a tournament are minimal. This paper studies the impact of winning a tournament on the productivity and effort choices of tournament participants in the post-tournament period.

We examine how winning the Fields Medal affects the post-medal productivity and research choices of mathematicians. The Fields Medal is the most prestigious award in mathematics, awarded every four years to mathematicians under the age of 40. Using archival data from the American Mathematical Society and the Mathematics Genealogy Project, we document the shape of the age-productivity profile of these exceptional mathematicians along a number of different dimensions, including the number of papers published, citations received, and students mentored. We find that the age-productivity profile of the Fields medalists and of the losing contenders is similar until the year in which a particular mathematician wins the Fields Medal (or does not win it). Remarkably, the rate of output of the Fields medalists declines noticeably in the post-medal period.



We also show that the medalists exhibit a far greater rate of cognitive mobility in the post-medal period, pursuing topics that are far less likely to be related to their pre-medal work. Because cognitive mobility is costly (e.g., additional time is required to prepare a paper in an unfamiliar field), the increased rate of mobility reduces the medalists' rate of output in the post-medal period. The data suggest that about half of the decreased productivity in the post-medal period can be attributed to the increased propensity for experimentation.

Hundreds of scientific and technical prizes are awarded around the world. Our evidence suggests that the post-prize productivity impact of winning a prestigious award can be substantial, affecting both the quantity and type of research the winners produce. Many Fields Medalists took Hermann Weyl's words to heart and "carried on as they began." This, however, was not the typical outcome. The data instead reveal that the increased opportunities provided by the Fields Medal, in fact, discouraged the recipients from continuing to produce the pure mathematics that the medal was awarded for, even while encouraging time-consuming investments in ever more distant locations in the space of ideas.

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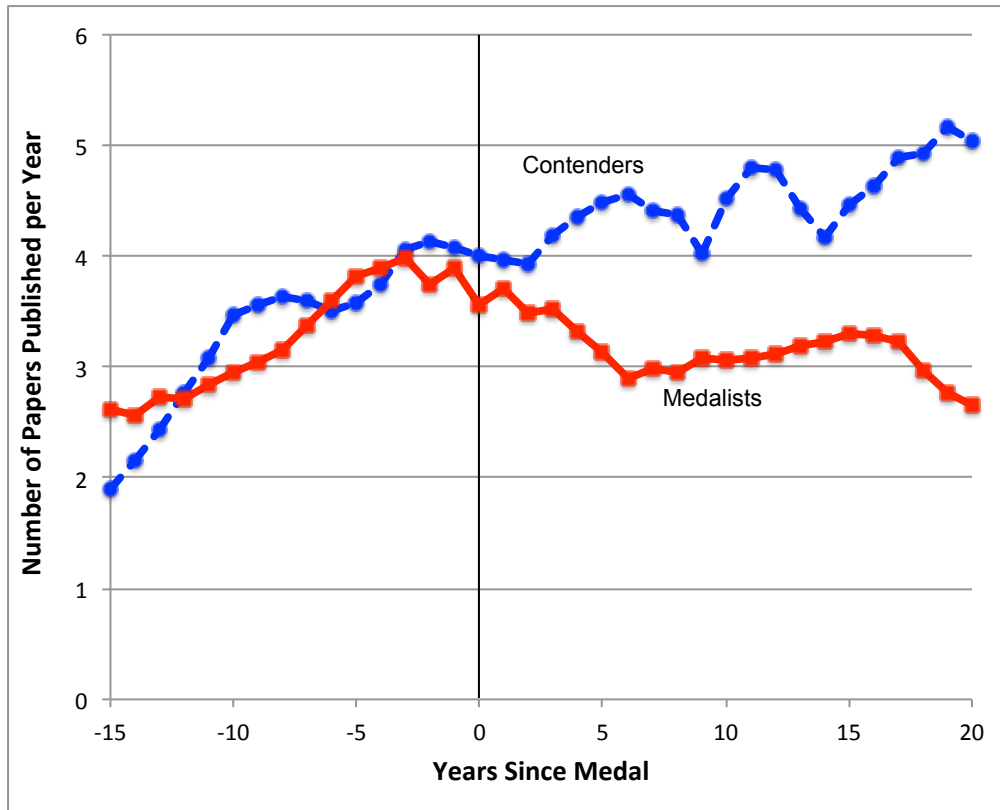
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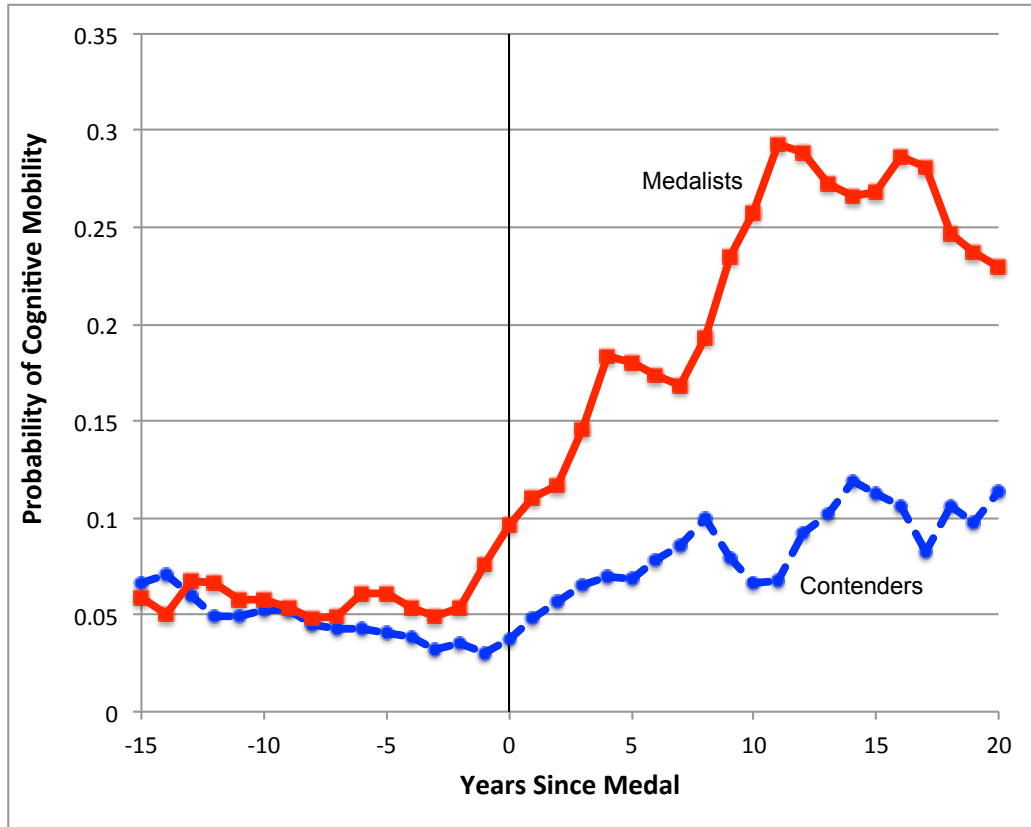
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**Figure 1. Average number of papers published annually by the Fields medalists and the contenders (3-year moving average)**



Notes: The group of “contenders” is composed of persons who were awarded at least one of six other mathematics prizes (the Abel, Wolf, Cole Algebra, Bôcher, Veblen, and Salem Prizes) and have above-median per-year citations during the eligibility period for the Fields Medal, but were not awarded the Fields Medal. We smooth out the trend by using a 3-year moving average centered on the middle year in the interval.

**Figure 2. The probability of cognitive mobility for the Fields medalists and the contenders (3-year moving average)**



Notes: The group of “contenders” is composed of persons who were awarded at least one of six other mathematics prizes (the Abel, Wolf, Cole Algebra, Bôcher, Veblen, and Salem Prizes) and have above-median per-year citations during the eligibility period for the Fields Medal, but were not awarded the Fields Medal. Cognitive mobility indicates if a paper published at any point during the mathematician’s career differs from the “baseline fields” in the eligibility period for the Fields Medal. The “baseline field” is defined by the set of the two modal fields and all related fields in which the mathematician published during the eligibility period. We smooth out the trend by using a 3-year moving average centered on the middle year in the interval.

**Table 1. Summary Statistics**

Variable:	All mathematicians	Fields medalists	Contenders with above-median citations	All contenders
Lifetime papers	31.8	116.5	126.4	106.4
Papers per year	0.9	3.1	3.6	2.7
20-39 years old	---	3.3	3.4	2.7
40-59 years old	---	2.9	4.1	3.0
Lifetime citations	93.6	2451.9	2213.5	1640.4
Citations per year	2.5	64.0	56.0	39.1
Citations per paper	1.8	21.0	17.5	15.6
Year of birth	---	1949.5	1953.7	1946.2
Year of first publication	1972.6	1972.6	1977.7	1970.8
Age at first publication	---	23.1	24.0	24.6
Deceased (percent)	---	10.6	14.0	20.9
Age at death	---	74.0	60.5	66.3
Number of mathematicians	72,140	47	43	86

Note: The summary statistics for “all mathematicians” are calculated using the group of mathematicians in the AMS database who had at least 20 years of experience before ending their publication career; i.e., those whose most recent publication is at least 20 years after their first publication. The group of “contenders” is composed of persons who were awarded at least one of six other mathematics prizes (the Abel, Wolf, Cole Algebra, Bôcher, Veblen, and Salem Prizes), but were not awarded the Fields Medal. The group of contenders with “above-median citations” is composed of the contenders who had above-median per-year citations during the eligibility period for the Fields Medal. The years included in the “papers per year” calculations across different age groups begin with the year of the first publication and end at age 59 or the year of death (whichever comes first).

**Table 2. Impact of the Fields Medal on the number of papers published per year**

Specification:	Sample of contenders			
	Contenders with above- median citations		All contenders	
	Post-medal period	Post-medal period × Fields Medal	Post-medal period	Post-medal period × Fields Medal
1. Number of papers	0.119 (0.548)	-1.378 (0.676)	-0.157 (0.291)	-0.918 (0.435)
2. Log number of papers	-0.074 (0.089)	-0.244 (0.105)	-0.093 (0.065)	-0.173 (0.090)
3. Papers, quantile regression	-0.421 (0.283)	-0.665 (0.256)	-0.314 (0.150)	-0.756 (0.161)
Number of papers:				
4. Contenders' post-medal period begins at age 36	0.160 (0.494)	-1.395 (0.656)	-0.166 (0.279)	-0.914 (0.435)
5. Contenders' post-medal period begins at age 40	0.266 (0.545)	-1.440 (0.681)	0.069 (0.286)	-0.987 (0.436)

Notes: Standard errors are reported in parentheses and are clustered at the individual level. The regressions using the sample of contenders with above-median citations have 3,269 observations (2,719 observations in the log papers regressions); the regressions using the sample of all contenders have 5,213 observations (4,109 observations in the log papers regressions). In rows 1-3, the contenders' post-medal period begins the year they are no longer eligible to receive the Fields Medal.

**Table 3. Impact of the Fields Medal on other annual measures of productivity**

Dependent variable:	Sample of contenders			
	Contenders with above- median citations		All contenders	
	Post-medal period	Post-medal period $\times$ Fields Medal	Post-medal period	Post-medal period $\times$ Fields Medal
1. Published at least one paper	0.030 (0.041)	-0.118 (0.047)	-0.002 (0.030)	-0.092 (0.044)
2. Number of citations	-1.146 (13.480)	-44.182 (14.645)	5.448 (8.826)	-44.493 (12.631)
3. Citations per paper	2.204 (3.351)	-11.000 (3.145)	2.560 (2.739)	-10.716 (2.866)
4. Probability of a “home run”	0.062 (0.035)	-0.075 (0.034)	0.054 (0.027)	-0.075 (0.031)
5. Probability of a “strikeout”	0.009 (0.024)	0.053 (0.028)	0.022 (0.021)	0.058 (0.026)
6. Number of mentees	-0.016 (0.078)	-0.126 (0.074)	-0.017 (0.054)	-0.137 (0.058)
7. Number of papers published by mentees	1.430 (3.194)	-3.981 (2.522)	0.883 (2.364)	-4.932 (1.828)
8. Number of citations generated by mentees	-12.676 (44.334)	-56.309 (37.395)	-17.963 (29.630)	-60.682 (29.787)

Notes: Standard errors are reported in parentheses and are clustered at the individual level. The regressions using the sample of contenders with above-median citations have 3,269 observations; the regressions using the sample of all contenders have 5,213 observations. The sample sizes for the regressions reported in rows 3-5 are 2,719 and 4,109 observations, respectively; and the sample sizes for the regressions reported in rows 5-7 are 2,999 and 4,797, respectively. A “home run” occurs when the number of citations per paper published in a given year is above the 99.5<sup>th</sup> percentile for all mathematicians in the AMS database; a “strikeout” occurs when the number of citations per paper published in a given year is zero.



**Table 4. Impacts of the Fields Medal Using ICM Plenary Speakers as Control Group**

Specification:	Sample of contenders			
	All plenary speakers		Plenary speakers with above-median citations	
	Post-medal period	Post-medal period $\times$ Fields Medal	Post-medal period	Post-medal period $\times$ Fields Medal
<b>A. Contenders invited to be plenary speakers at any age</b>				
1. Number of papers	0.096 (0.189)	-1.076 (0.356)	-0.091 (0.291)	-0.955 (0.432)
2. Log number of papers	-0.014 (0.043)	-0.239 (0.080)	-0.059 (0.059)	-0.183 (0.088)
3. Citations	8.998 (7.305)	-50.058 (9.698)	3.749 (11.742)	-47.642 (14.952)
4. Citations per paper	1.217 (2.011)	-10.950 (2.709)	0.257 (3.035)	-10.012 (4.013)
5. Probability of a “home run”	0.030 (0.019)	-0.078 (0.029)	0.034 (0.029)	-0.066 (0.032)
6. Probability of a “strikeout”	0.006 (0.152)	0.073 (0.024)	0.011 (0.018)	0.048 (0.023)
<b>B. Contenders invited to be plenary speakers by age 44</b>				
1. Number of papers	0.039 (0.338)	-0.906 (0.448)	-0.029 (0.584)	-1.130 (0.685)
2. Log number of papers	-0.063 (0.067)	-0.161 (0.089)	-0.092 (0.095)	-0.188 (0.107)
3. Citations	0.587 (10.427)	-39.978 (12.928)	-8.096 (14.276)	-37.961 (15.344)
4. Citations per paper	0.386 (2.756)	-8.993 (2.876)	0.672 (3.912)	-9.019 (3.468)
5. Probability of a “home run”	0.040 (0.026)	-0.065 (0.038)	0.060 (0.035)	-0.072 (0.035)
6. Probability of a “strikeout”	0.011 (0.017)	0.030 (0.026)	0.023 (0.024)	0.023 (0.025)

Notes: Standard errors are reported in parentheses and are clustered at the individual level. The sample sizes for the regressions reported in rows 1 and 3 of Panel A are 9,011 and 5,133 observations, respectively; and the sample sizes for the regressions reported in all other rows are 7,505 and 4,396, respectively. The sample sizes for the regressions reported in rows 1 and 3 of Panel B are 4,745 and 3,171 observations, respectively; and the sample sizes for the regressions reported in all other rows are 3,966 and 2,655, respectively. A “home run” occurs when the number of citations per paper published in a given year is above the 99.5<sup>th</sup> percentile for all mathematicians in the AMS database; a “strikeout” occurs when the number of citations per paper published in a given year is zero.

**Table 5. Impact of the Fields Medal on the probability of cognitive mobility**

	Baseline field defined using two modes		Baseline field defined using one mode	
	Post-medal period	Post-medal period × Fields Medal	Post-medal period	Post-medal period × Fields Medal
<u>Sample and specification:</u>				
A, Using sample of contenders with above-median citations				
1. Baseline = eligibility period	-0.049 (0.031)	0.163 (0.036)	-0.065 (0.038)	0.213 (0.039)
2. Baseline = first 3 years	-0.064 (0.038)	0.132 (0.039)	-0.049 (0.043)	0.135 (0.042)
3. Baseline = first 5 years	-0.066 (0.033)	0.158 (0.036)	-0.025 (0.036)	0.147 (0.039)
B. Using sample of all contenders				
1. Baseline = eligibility period	-0.016 (0.025)	0.146 (0.033)	-0.020 (0.030)	0.183 (0.040)
2. Baseline = first 3 years	-0.042 (0.030)	0.121 (0.036)	-0.034 (0.033)	0.134 (0.040)
3. Baseline = first 5 years	-0.029 (0.028)	0.135 (0.034)	-0.010 (0.030)	0.144 (0.038)

Notes: Standard errors are reported in parentheses and are clustered at the individual level. The “baseline field” is defined by the set of the (one or two) modal fields and all related fields in which the mathematician published during the baseline period (either the entire eligibility period, the first 3 years, or the first 5 years of his career). The dependent variable is a cognitive mobility indicator set to unity if the field of publication for each paper during the mathematician’s career is not in the baseline field. The regressions in Panel A have 10,911 observations; the regressions in Panel B have 14,628 observations.

**Table 6. Cognitive mobility and the duration of the preparation spell**

	Baseline field defined using two modes		Baseline field defined using one mode	
	Fields medalist	Contender	Fields medalist	Contender
<u>Sample and specification:</u>				
A, Using sample of contenders with above-median citations				
1. Baseline = eligibility period	0.213 (0.033)	0.229 (0.038)	0.173 (0.043)	0.212 (0.035)
2. Baseline = first 3 years	0.202 (0.035)	0.196 (0.039)	0.183 (0.036)	0.164 (0.039)
3. Baseline = first 5 years	0.193 (0.030)	0.201 (0.034)	0.212 (0.036)	0.183 (0.033)
B. Using sample of all contenders				
1. Baseline = eligibility period	0.211 (0.032)	0.263 (0.035)	0.171 (0.040)	0.214 (0.036)
2. Baseline = first 3 years	0.200 (0.036)	0.208 (0.039)	0.181 (0.037)	0.192 (0.035)
3. Baseline = first 5 years	0.193 (0.031)	0.224 (0.033)	0.209 (0.036)	0.194 (0.034)

Notes: Standard errors are reported in parentheses and are clustered at the individual level. The length of the preparation is the length of time elapsed (in years) between any two consecutive papers in a mathematician's career. The "baseline field" is defined by the set of the (one or two) modal fields and all related fields in which the mathematician published during the baseline period (either the entire eligibility period, the first 3 years, or the first 5 years of his career). The dependent variable is a cognitive mobility indicator set to unity if the field of publication for each paper during the mathematician's career is not in the baseline field. The regressions in Panel A have 10,821 observations; the regressions in Panel B have 14,495 observations.

### Appendix Table. List of Fields medalists and contenders

Anantharaman, Nalini, <sup>S</sup>	Jones, Peter, <sup>S</sup>	Rosenlicht, Maxwell, <sup>C</sup>
Arnold, Vladimir, <sup>W</sup>	Journé, Jean-Lin, <sup>S</sup>	Roth, Klaus, <sup>F</sup>
Artin, Michael, <sup>W</sup>	Keller, Joseph, <sup>W</sup>	Sato, Mikio, <sup>W</sup>
Aschbacher, Michael, <sup>W,C</sup>	Kenig, Carlos, <sup>S</sup>	Schoen, Richard, <sup>B</sup>
Astala, Kari, <sup>S</sup>	Kirby, Robion, <sup>V</sup>	Schramm, Oded, <sup>S</sup>
Atiyah, Michael, <sup>F,A</sup>	Klartag, Boáz, <sup>S</sup>	Seidel, Paul, <sup>V</sup>
Avila, Artur, <sup>S</sup>	Kontsevich, Maxim, <sup>F</sup>	Serre, Jean-Pierre, <sup>F,A,W</sup>
Baker, Alan, <sup>F</sup>	Konyagin, Sergei, <sup>S</sup>	Shelah, Saharon, <sup>W</sup>
Beckner, William, <sup>S</sup>	Körner, Thomas, <sup>S</sup>	Shishikura, Mitsuhiro, <sup>S</sup>
Bombieri, Enrico, <sup>F</sup>	Lacey, Michael, <sup>S</sup>	Simons, James, <sup>V</sup>
Borcherds, Richard, <sup>F</sup>	Lafforgue, Laurent, <sup>F</sup>	Sinaï, Yakov, <sup>W</sup>
Bott, Raoul, <sup>W</sup>	Lang, Serge, <sup>C</sup>	Singer, Isadore, <sup>A</sup>
Bourgain, Jean, <sup>F,S</sup>	Langlands, Robert, <sup>W</sup>	Smale, Stephen, <sup>F,W,V</sup>
Brown, Morton, <sup>V</sup>	Lax, Peter, <sup>A,W</sup>	Smirnov, Stanislav, <sup>F,S</sup>
Caffarelli, Luis, <sup>W,B</sup>	Lindenstrauss, Elon, <sup>F,S</sup>	Soundararajan, Kannan, <sup>S</sup>
Calderón, Alberto, <sup>W</sup>	Lions, Pierre-Louis, <sup>F</sup>	Stallings, John, <sup>C</sup>
Carleson, Lennart, <sup>A,W</sup>	Lovász, László, <sup>W</sup>	Stein, Elias, <sup>W</sup>
Chandra, Harish, <sup>C</sup>	Lusztig, George, <sup>C</sup>	Sullivan, Dennis, <sup>W,V</sup>
Cohen, Paul, <sup>F,B</sup>	Margulis, Grigory, <sup>F,W</sup>	Swan, Richard, <sup>C</sup>
Connes, Alain, <sup>F</sup>	Mazur, Barry, <sup>V</sup>	Szemerédi, Endre, <sup>A</sup>
Dahlberg, Björn, <sup>S</sup>	McMullen, Curtis, <sup>F,S</sup>	Tao, Terence, <sup>F,B,S</sup>
David, Guy, <sup>S</sup>	Melrose, Richard, <sup>B</sup>	Tataru, Daniel, <sup>B</sup>
de Jong, Aise Johan, <sup>C</sup>	Meyer, Yves, <sup>S</sup>	Tate, John, <sup>A,W</sup>
Deligne, Pierre, <sup>F,A,W</sup>	Milnor, John, <sup>F,A,W</sup>	Taubes, Clifford, <sup>V</sup>
Donaldson, Simon, <sup>F</sup>	Montgomery, Hugh, <sup>S</sup>	Thiele, Christoph, <sup>S</sup>
Drinfeld, Vladimir, <sup>F</sup>	Mori, Shigefumi, <sup>F,C</sup>	Thom, René, <sup>F</sup>
Faltings, Gerd, <sup>F</sup>	Moser, Jürgen, <sup>W</sup>	Thompson, John, <sup>F,A,W,C</sup>
Fefferman, Charles, <sup>F,B,S</sup>	Mostow, George, <sup>W</sup>	Thurston, William, <sup>F,V</sup>
Feit, Walter, <sup>C</sup>	Mumford, David, <sup>F,W</sup>	Tian, Gang, <sup>V</sup>
Freedman, Michael, <sup>F,V</sup>	Naor, Assaf, <sup>B,S</sup>	Tits, Jacques, <sup>A,W</sup>
Furstenberg, Hillel, <sup>W</sup>	Nazarov, Fedor, <sup>S</sup>	Tolsa, Xavier, <sup>S</sup>
Gowers, Timothy, <sup>F</sup>	Ngô, Báo Châu, <sup>F</sup>	Varadhan, S.R., <sup>A</sup>
Green, Ben, <sup>S</sup>	Nikishin, Evgeniï, <sup>S</sup>	Varopoulos, Nicholas, <sup>S</sup>
Griffiths, Phillip, <sup>W</sup>	Nirenberg, Louis, <sup>B</sup>	Venkatesh, Akshay, <sup>S</sup>
Gromov, Mikhael, <sup>A,W,V</sup>	Novikov, Sergei, <sup>F,W</sup>	Villani, Cédric, <sup>F</sup>
Grothendieck, Alexander, <sup>F</sup>	Okounkov, Andrei, <sup>F</sup>	Voevodsky, Vladimir, <sup>F</sup>
Hacon, Chistopher, <sup>C</sup>	Ornstein, Donald, <sup>B</sup>	Volberg, Alexander, <sup>S</sup>
Herman, Michael-Robert, <sup>S</sup>	Ozsváth, Peter, <sup>V</sup>	Werner, Wendelin, <sup>F</sup>
Hironaka, Heisuke, <sup>F</sup>	Perelman, Grigori, <sup>F</sup>	Wiles, Andrew, <sup>W</sup>
Hirzebruch, Friedrich, <sup>W</sup>	Petermichl, Stefanie, <sup>S</sup>	Witten, Edward, <sup>F</sup>
Hochster, Melvin, <sup>C</sup>	Piatetski-Shapiro, Ilya, <sup>W</sup>	Wolff, Thomas, <sup>S</sup>
Hörmander, Lars, <sup>F,W</sup>	Pichorides, Stylianos, <sup>S</sup>	Wooley, Trevor, <sup>S</sup>
Hunt, Richard, <sup>S</sup>	Pisier, Giles, <sup>S</sup>	Yau, Shing-Tung, <sup>F,W,V</sup>
Jones, Vaughan, <sup>F</sup>	Quillen, Daniel, <sup>F,C</sup>	Yoccoz, Jean-Christophe, <sup>F,S</sup>
		Zelmanov, Efim, <sup>F</sup>

Notes: The superscripts indicate the prize awarded to the mathematician; F = Fields Medal; A = Abel Prize; W = Wolf Prize; C = Cole Algebra Prize; B = Bôcher Prize; V = Veblen Prize; and S = Salem Prize.