Quantum field theory on the light-front

Theory group seminar, UiB, 9 Feb 2023

Old story

- In QM, the dynamical evolution satisfies the Schrödinger equation
$$i\frac{\partial}{\partial t}|\psi(t)\rangle = H|\psi(t)\rangle$$

- t is usually the standard time variable ($t \equiv x^0$ in Minkowski space-time)

- but other foliations can be realized!

IEWS OF MODERN PHYSICS VOLUME 21, NUMBER 3 JULY, 1949

Forms of Relativistic Dynamics

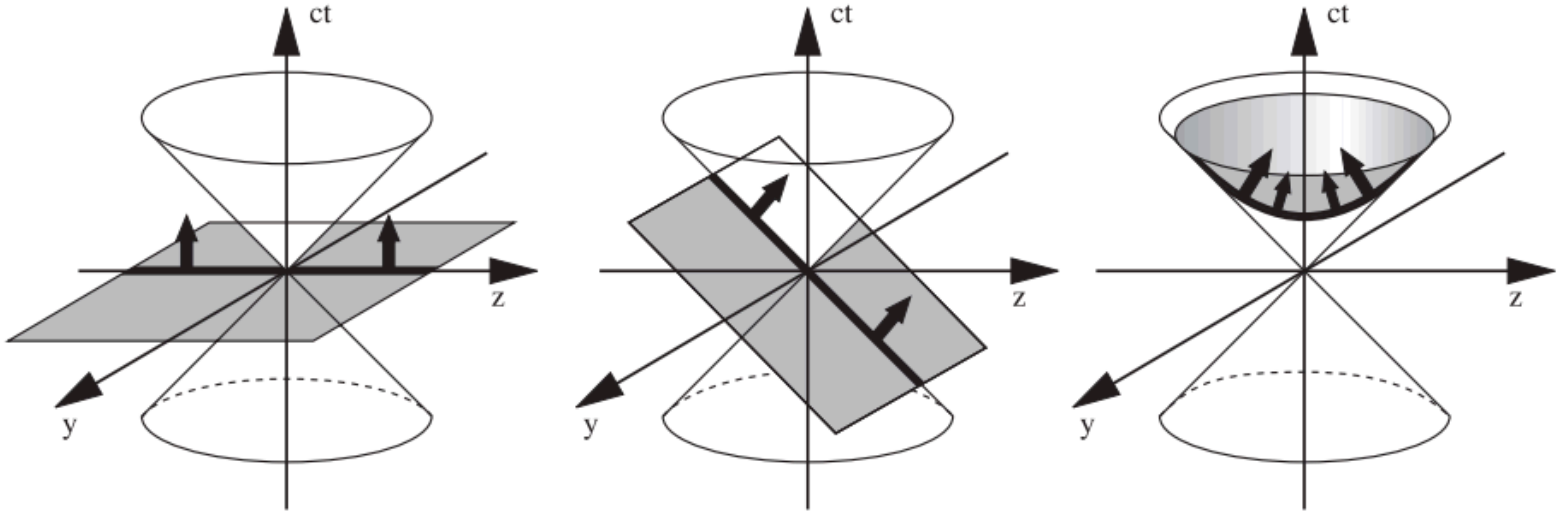
P. A. M. Dirac
St. John's College, Cambridge, England

For the purposes of atomic theory it is necessary to combine the restricted principle of relativity with the Hamiltonian formulation of dynamics. This combination leads to the appearance of ten fundamental quantities for each dynamical system, namely the total energy, the total momentum and the 6-vector which has three components equal to the total angular momentum. The usual form of dynamics expresses everything in terms of dynamical variables at one instant of time, which results in specially simple expressions for six or these ten, namely the components of momentum and of angular momentum. There are other forms for relativistic dynamics in which others of the ten are specially simple, corresponding to various sub-groups of the inhomogeneous Lorentz group. These forms are investigated and applied to a system of particles in interaction and to the electromagnetic field.

5. THE FRONT FORM

Consider the three-dimensional surface in space-time formed by a plane wave front advancing with the velocity of light. Such a surface will be called a *front* for brevity. An example of a front is given by the equation

$$u_0 - u_3 = 0. \tag{26}$$



The instant form

$$\tilde{x}^0 = ct,\quad \tilde{x}^1 = x,\quad \tilde{x}^2 = y,\quad \tilde{x}^3 = z$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The front form

$$r^+ = ct + z,\quad r^1 = x,\quad r^2 = y,\quad r^- = ct - z$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$$

The point form

$$\begin{aligned} \tilde{x}^0 &= \tau &,\quad ct &= \tau \cosh\omega \\ \tilde{x}^1 &= \omega &,\quad x &= \tau \sinh\omega \sin\theta \cos\phi \\ \tilde{x}^2 &= \theta &,\quad y &= \tau \sinh\omega \sin\theta \sin\phi \\ \tilde{x}^3 &= \phi &,\quad z &= \tau \sinh\omega \cos\theta \end{aligned}$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\tau^2 & 0 & 0 \\ 0 & 0 & -\tau^2 \sinh^2\omega & 0 \\ 0 & 0 & 0 & -\tau^2 \sinh^2\omega \sin^2\theta \end{pmatrix}$$

8-2004
8701A1

Hadronic eigenvalue problem

S. Brodsky, H-C Pauli, S. Pinsky Phys.Rept.301:299-486,1998

- finding bound states of $H_{\mathrm{LF}} = P^+P^- + P_\perp^2$
- P^+ (operator) has only positive (eigen)values
- basis $|\Psi\rangle = |\Psi; M, P^+, P_\perp, S^2, S_z; h\rangle$ contains quantum numbers for each hadron *h*
- Fock state basis: $|\Psi> = \sum_n \int \mathrm{d}[\mu_n]\,|\mu_n\rangle\,\langle\mu_n|\Psi\rangle$
- For practical computations, only a fixed set of Fock states are considered.

$$H_{\mathrm{LF}}|\Psi\rangle = M_h^2|\Psi\rangle$$

$$\begin{array}{llcrl}
n=0: & |0\rangle\ , & & & \\
n=1: & \left|q\bar{q}: k_i^+, \vec{k}_{\perp i}, \lambda_i\right\rangle & = & b^\dagger(q_1)\, d^\dagger(q_2) & |0\rangle \\
n=2: & \left|q\bar{q}g: k_i^+, \vec{k}_{\perp i}, \lambda_i\right\rangle & = & b^\dagger(q_1)\, d^\dagger(q_2)\, a^\dagger(q_3) & |0\rangle \\
n=3: & \left|gg: k_i^+, \vec{k}_{\perp i}, \lambda_i\right\rangle & = & a^\dagger(q_1)\, a^\dagger(q_2) & |0\rangle \\
\vdots & \vdots & \vdots & \vdots & |0\rangle
\end{array}$$

The operators $b^\dagger(q)$,$d^\dagger(q)$ and $a^\dagger(q)$ create bare leptons (electrons or quarks), bare anti-leptons (positrons or antiquarks) and bare vector bosons (photons or gluons).

Time-dependent problems

Xingbo Zhao, Anton Ilderton, Pieter Maris, James P. Vary Phys. Rev. D 88 (2013) 065014

- powerful method to deal with time-dependent background fields

- lot of use in strong laser fields - can also probe beyond SM physics!

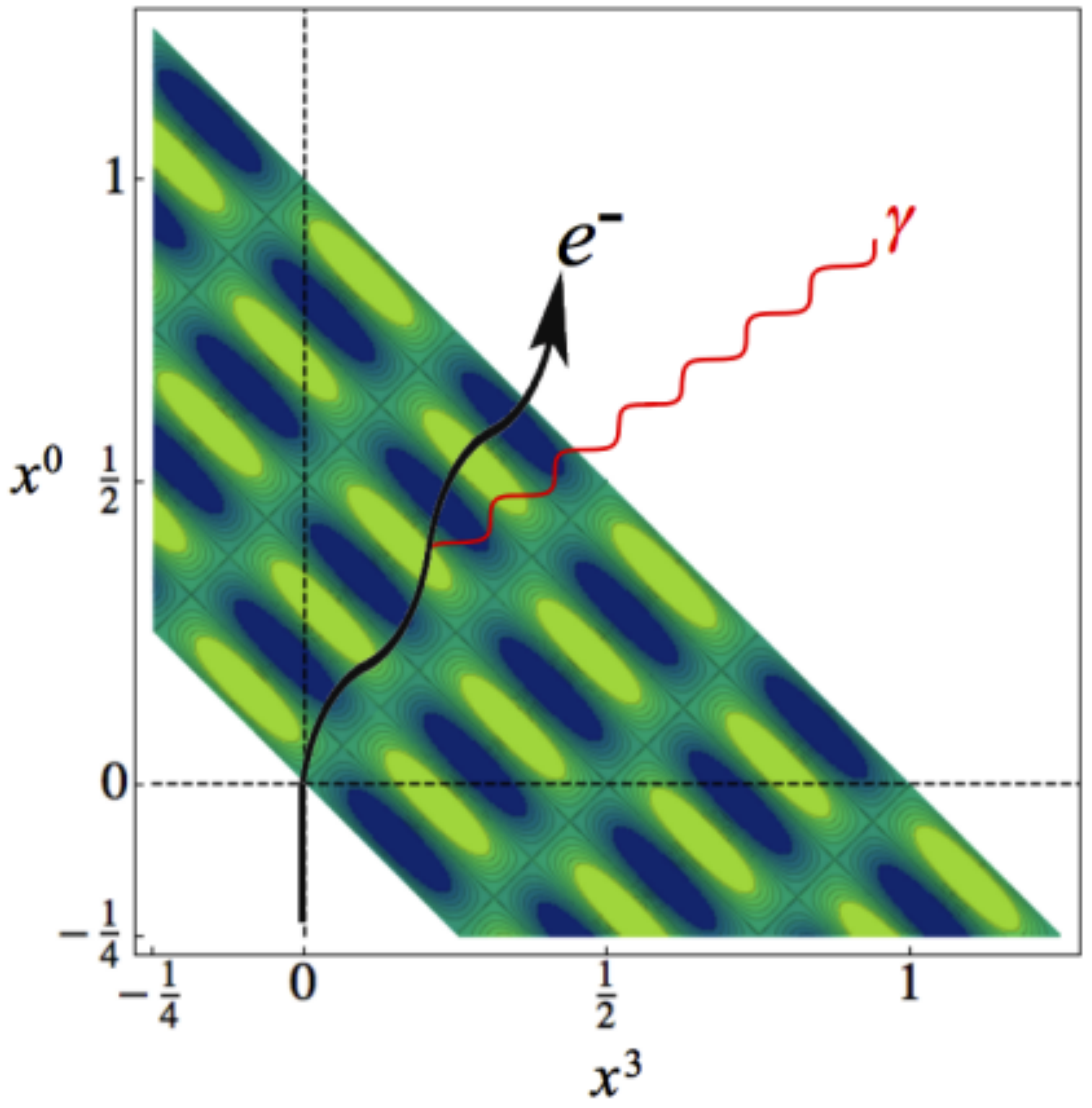
$$i\frac{\partial}{\partial x^+}|\,\psi;x^+\,\rangle = \frac{1}{2}P^-(x^+)|\,\psi;x^+\,\rangle$$

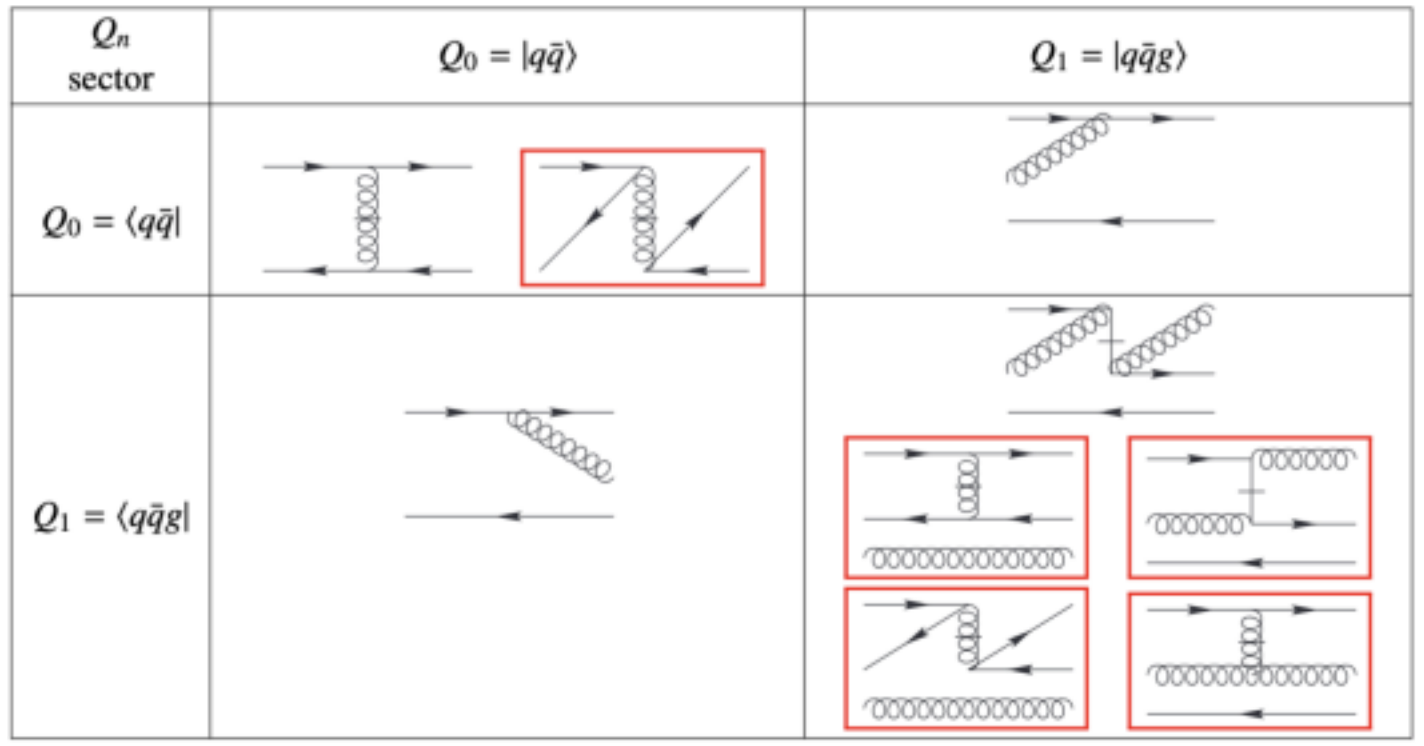
$$P^-(x^+) = P^-_{\rm QED} + V(x^+)$$

$$|\,\psi;x^+\,\rangle_I = e^{\frac{i}{2}P^-_{\rm QED}x^+}|\,\psi;x^+\,\rangle$$

$$V_I(x^+) = e^{\frac{i}{2}P^-_{\rm QED}x^+}V(x^+)e^{-\frac{i}{2}P^-_{\rm QED}x^+}$$

$$|\,\psi;x^+\,\rangle_I = \mathcal{T}_+\exp\Big(-\frac{i}{2}\int_0^{x^+}V_I\Big)|\,\psi;0\,\rangle_I$$





Applications in nuclear physics

Gerald A. Miller Prog.Part.Nucl.Phys.45:83-155,2000

- typically interested in scattering processes
- deep-inelastic scattering $e + A \rightarrow e' + A'$ (scattering off cold nuclear matter)
- propagation of jets in medium
  - the background in QGP

$$d\sigma \sim \sum_f \int \frac{d^3 p_f}{E_f} \int d^4 p \, \delta(p^2 - M^2) \delta^{(4)}(q + p_i - p_f - p) |\langle p, f \mid J(q) \mid i \rangle \, |^2,$$

More details in:
https://arxiv.org/pdf/2212.05100.pdf
https://arxiv.org/abs/hep-ph/9612244

Hydrogen Atom in Relativistic Motion https://arxiv.org/abs/hep-ph/0411208