Master Project

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Strichartz estimates for nonlinear Schrödinger equations on compact manifolds

Strichartz estimates are a type of refined Sobolev embeddings obtained in the context of linear dispersive equations. They have been a key tool in the study of nonlinear dispersive equations in the last decades (see for instance [2, 3]).

The first goal of this master project is to study the Strichartz estimates for the linear Schrödinger equation on compact manifolds

$$\begin{cases} i\partial_t u + \Delta u = 0\,, \\ u(\cdot,0) = u_0\,, \end{cases} \tag{1}$$

where $u = u(x,t)$ is a complex value function, $t \in \mathbb{R}$, $x \in M$, $t \in \mathbb{R}$, (M,g) is a Riemannian compact manifold of dimension $d \geq 1$ and $\Delta = \Delta_g$ is the corresponding Laplace-Beltrami operator on M.

It has been proved in [1, 4] that for any (p,q) satisfying $2 \leq p$, $q < +\infty$ and $2/p + d/q = d/2$,

$$\|u\|_{L^p(I,L^q(M))} \leq C(I)\|u_0\|_{H^{1/p}(M)}\,,$$

for any finite temporal interval I. Observe that, contrary to the $\mathbb{R}^d$ case, one has to admit loss of derivative of $1/p$ in the estimate. We will follow the proof in [1], which relies on dispersive estimates on small time intervals, whose length depends on the size of the space frequency of the initial data.

As an application, still following [1], we will prove some well-posedness results for the cubic nonlinear Schrödinger equation on compact manifolds

$$\begin{cases} i\partial_t u + \Delta u = |u|^2 u\,, \\ u(\cdot,0) = u_0\,, \end{cases} \tag{2}$$

in dimension $d = 2$ and $d = 3$.

Finally, if times allows, we will try to obtain new well-posedness results for other nonlinear dispersive equations on manifolds.

References

[1] N. Burq, P. Gérard and N. Tzvetkov, *Strichartz inequalities and the nonlinear Schrödinger equation on compact manifolds*, Amer. J. Math., **126** (2004), 569-605.

[2] F. Linares, and G. Ponce, Introduction to Nonlinear Dispersive Equations, (second edition), Springer, New York, 2015

[3] M. Keel and T. Tao, *Endpoint Strichartz estimates,* Amer. J. Math., **120** (1998), 955-980.

[4] G. Staffilani and D. Tataru, *Strichartz estimates for a Schrödinger operator with nonsmooth coefficients,* Comm. Part. Diff. Equ., **27** (2002), 1337-1372.