

# PhD Project Description: Hybrid Coalition Logic

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**Abstract.** The aim of this project is to extend Pauly’s Coalition Logic with nominals and modalities from hybrid modal logic in order to significantly increase its expressive power, and to study the resulting Hybrid Coalition Logic. Coalition Logic is one of the main frameworks for formalizing reasoning about coalitional strategic ability in artificial intelligence and computational social choice. While several researchers have extended the expressive power of basic Coalition Logic, I am not aware of any works on “hybridizing” Coalition Logic, which is surprising since “hybridization” is one of the most successful approaches to extend the expressive power of normal modal logics.

## 1 Introduction

Logical formalization of reasoning forms one of the foundations of artificial intelligence (AI) and computer science, providing the basics for knowledge representation and reasoning and for formal specification and verification. Most modern approaches are based on modal logic. Of considerable interest in AI and multi-agent system (MAS) in recent years have been logics formalizing reasoning about strategic ability. The main frameworks are Alternating-time Temporal Logic (ATL) [6] and Coalition Logic [1].

Coalition Logic is a modal logic for reasoning about what groups of agents can bring about strategically by collective action. In other words, Coalition Logic is reasoning about so-called  $\alpha$ -effectivity in strategic game structures and the modality  $\langle\langle C \rangle\rangle\phi$  means the coalition  $C$  is effective for  $\phi$  –  $C$  has a strategy to ensure that  $\phi$  will be true no matter what other agents do. In this logic, there are five important properties: Liveness, Safety, N-maximality, Outcome-monotonicity, Superadditivity. In [1], Pauly proved that the five properties can characterize  $\alpha$ -effectivity functions, that is,  $E_G^\alpha = E$  and its completeness. Unfortunately, in [5], Goranko, Jamroga and Turrini proved that the five properties can not characterize  $\alpha$ -effectivity functions, because in any strategic game form, empty coalition can force the sets that contain all possible outcomes in the game. However, the result can not affect the completeness of Coalition Logic. From the viewpoint of semantics, Coalition Logic uses neighborhood structures to describe the outcomes which groups of agents can force in game-like situation. Coalition Logic provides a link between computer science, artificial intelligence and the social sciences [7], in particular in computational social choice and algorithmic game theory [8]. ATL can be seen as an extension of Coalition Logic with temporal modalities [9]. Considerable work in the AI and MAS communities in recent years has focussed on extending the expressive power of these logics, e.g. in order to reason about knowledge and belief [10–12]

or adding quantifiers [13, 14] and analyzing the logical and computational properties of the resulting logics. Of particular interest has been axiomatic completeness and the trade-offs between expressive power and computational complexity.

In normal modal logics [4], a well-known and very powerful way to extend the expressive power is to add nominals and other hybrid operators – leading us into the field of hybrid modal logics. Hybrid Logic was built based on basic normal modal logic by adding nominals, which are propositional symbols of a new sort interpreted in a restricted way that enables reference to individual points in a Kripke model. The history of Hybrid Logic goes back to the philosopher Arthur Prior’s work in the 1960s. In fact, Hybrid Logic has different sorts, like  $\mathcal{H}$ ,  $\mathcal{H}(@)$ ,  $\mathcal{H}(E)$  and  $\mathcal{H}(@, \downarrow)$ . However, so far almost all research on hybrid modal logics, with a few exceptions [2, 3], have been on normal modal logics. Coalition Logic, on the other hand, is not a normal modal logic – the completeness result mentioned above is with respect to neighborhood frames and not with respect to Kripke frames used in normal modal logics. The goal of this paper is to combine Coalition Logic with hybrid logic and study the logical and computational properties of the resulting Hybrid Coalition Logic, in order to significantly increase the expressive power and thus its usefulness for knowledge representation and reasoning and automated verification. This is nontrivial, as hybridizations of non-normal modal logics is not well understood.

## 2 Background

### 3 $\mathcal{HCL}(@)$

### 4 $\mathcal{HCL}(E)$

## References

- [1] Pauly, M. A modal logic for coalitional power in games. *Journal of logic and computation*, 12(1), 149-166, 2002.
- [2] Sano, K. Topological Pure Completeness for Hybrid Logics without the Global Modality. In *The 40th MLG Meeting*.
- [3] ten Cate, B., Litak, T. Topological perspective on the hybrid proof rules. *Electronic Notes in Theoretical Computer Science*, 174(6), 79-94, 2007.
- [4] Blackburn, P., De Rijke, M., Venema, Y. *Modal logic: graph. Darst* (Vol. 53). Cambridge University Press, 2002.
- [5] Goranko, V., Jamroga, W., Turrini, P. Strategic games and truly playable effectivity functions. *Autonomous Agents and Multi-Agent Systems*, 26(2), 288-314, 2013.
- [6] Alur, R., Henzinger, T. A., Kupferman, O. Alternating-time temporal logic. *Journal of the ACM (JACM)*, 49(5), 672-713, 2002.
- [7] Thomas Agotnes. *Coalition logic*. In Byron Kaldis, editor, *Encyclopedia of Philosophy and the Social Sciences*. SAGE, 2013.
- [8] Ågotnes, T., van der Hoek, W., Wooldridge, M. Reasoning about coalitional games. *Artificial Intelligence*, 173(1), 45-79, 2009.

- [9] Emerson, E. A. Temporal and modal logic. In *Formal Models and Semantics* (pp. 995-1072). Elsevier, 1990.
- [10] Ågotnes, T., Alechina, N. Epistemic coalition logic: completeness and complexity. In *Proceedings of the 11th International Conference on Autonomous Agents and Multiagent Systems-Volume 2* (pp. 1099-1106). International Foundation for Autonomous Agents and Multiagent Systems, June, 2012.
- [11] Ågotnes, T., Alechina, N. (2016). Coalition logic with individual, distributed and common knowledge. *Journal of Logic and Computation*, 2016.
- [12] Ågotnes, T., Wáng, Y. N. (2017). Resolving distributed knowledge. *Artificial Intelligence*, 252, 1-21, 2017.
- [13] Ågotnes, T., van der Hoek, W., Wooldridge, M. Quantifying over coalitions in epistemic logic. In *Proceedings of the 7th international joint conference on Autonomous agents and multiagent systems-Volume 2* (pp. 665-672). International Foundation for Autonomous Agents and Multiagent Systems, May, 2008.
- [14] Ågotnes, T., Van Der Hoek, W., Wooldridge, M. Quantified coalition logic. *Synthese*, 165(2), 269-294, 2008.