Theoretical and practical algorithms for CSP parameterized by the number of variables with value-independent constraints

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Outline

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Workflow Satisfiability Problem (WSP)

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(basic) WSP instance: a set $U = \{u_1, u_2, u_3, u_4\}$ of users and a set S of six steps as follows.

s_1 create purchase order	s_2 approve purchase order
s_3 sign goods received note	s_4 countersign goods received note
s_5 create payment	s_6 approve payment

Workflow Satisfiability Problem (WSP), cont'd

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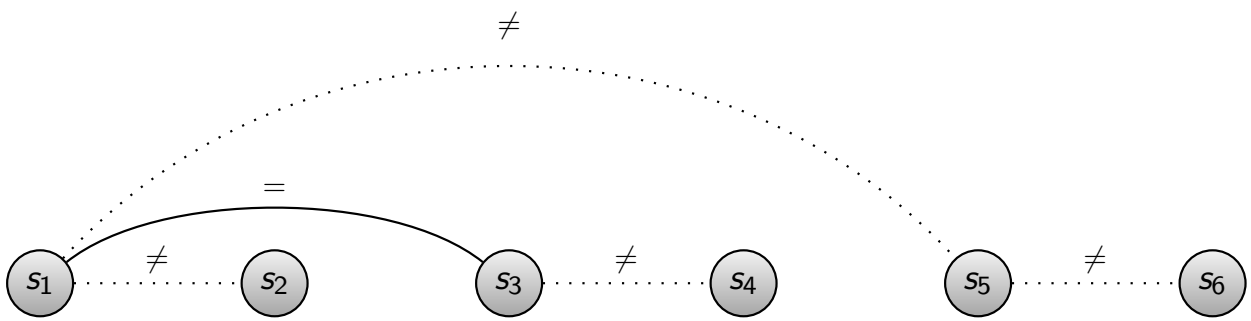
Constraints and authorizations:



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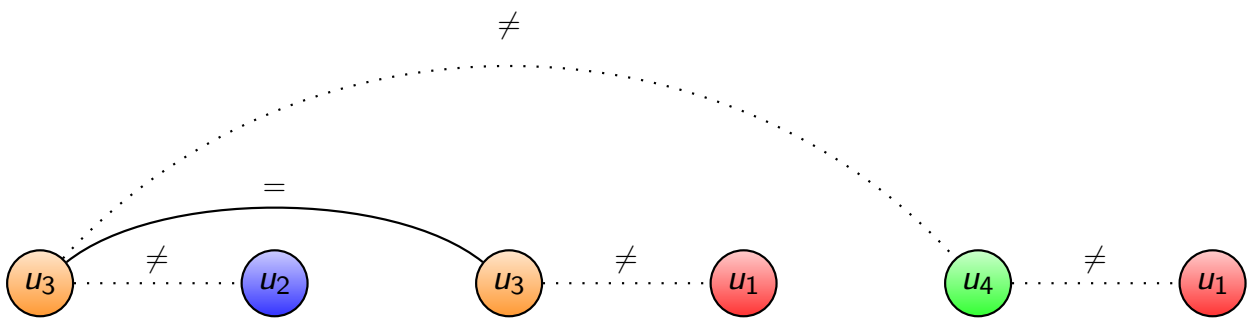

Authorized Users

u_3	u_2	u_3	u_1	u_1	u_1
u_1	u_3	u_4	u_3	u_3	u_4
u_2				u_4	

Is there a solution? Yes.

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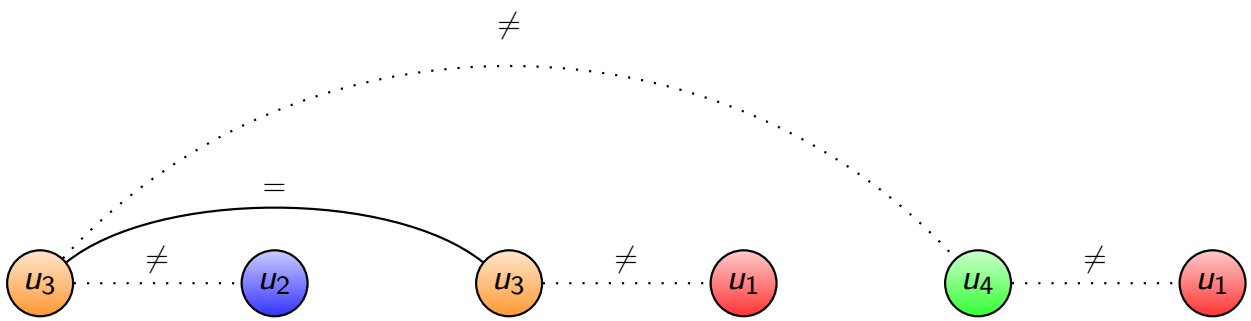

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u_1	u_3	u_4	u_3	u_3	u_4
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Is there a solution? Yes. It's an instance of List Coloring.

Definition of basic WSP

- Instance $W = (S, U, C)$, where S and U are sets of steps and users, C is a family of non-unary constraints with variables S and domain U. Unary constraints (authorizations): $A(s_i) = U'_i \subseteq U$.

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- basic WSP = CSP, where CSP variables = WSP steps, CSP values = WSP users. We'll talk about WSP.

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- Improved and generalized by Crampton, GG and Yeo (ACM Trans. Inf. Syst. Secur., 2013) to regular (undefined here) non-unary constraints with $O^*(2^k)$-algorithm. (Incl.Excl.)

Outline

User-Independent Constraints

- Cohen, Crampton, Gagarin, GG and Jones (J. AI Res., 2014): constraint c is user-independent (UI) if for every plan $\pi : S \to U$ satisfying c and permutation θ of U, plan $\pi' : S \to U$ also satisfies c, where for each $s \in S$, $\pi'(s) = \theta(\pi(s))$.

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- Counting constraints $(r, Q, \leq)$ and $(r, Q, \geq)$ are UI. $(r, Q, \leq)$ is satisfied by π if $|\pi(Q)| \leq r$ (confidentiality) and $(r, Q, \geq)$ is satisfied by π if $|\pi(Q)| \geq r$ (diversity). Example: AllDiff(Q)=$(|Q|, Q, \geq)$.

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- All the non-unary constraints defined in the 2004 American National Standards Institute Role Based Access Control standard are UI.

Results for Non-unary User-Independent Constraints (Unary Constraints are Arbitrary)

Cohen, Crampton, Gagarin, GG and Jones (JAIR 2014):

- k-WSP with UI non-unary constraints can be solved in time $O^*(2^{k \log k})$.

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- BFS-like user-iterative algorithm of exponential space.

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- Karapetyan, Gagarin and GG (FAW 2015): a backtracking $O^*(2^{k \log k})$-time poly-space algorithm for k-WSP with UI non-unary constraints, much faster in practice. Further improved by Karapetyan, Parks, GG and Gagarin (JAIR 2019, ta). [arXiv:1604.05636; Will be discussed in detail.]

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- Gutin and Wahlström (IPL 2016): k-WSP with UI non-unary constraints cannot be solved in time $O^*(c^{k \log k})$ for any $c < 2$ unless SETH fails.

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- Complete patterns are representatives of plans.
- Basic Algorithm has two parts: (1) checking UI non-unary constraints, (2) checking unary constraints.
- Improved Algorithm operates by generating partial patterns (corresponding to functions from subsets of S to U) gradually leading to complete patters unless some constraint is unsatisfied in which case backtracking.

Basic Algorithm: Part 1

- Since non-unary constraints are UI and up to k users can be used for steps $s_1, \ldots, s_k$, introduce complete patterns: tuples $(p_1, \ldots, p_k)$, where $p_i \in [k]$, if $p_i = p_j$ then s_i, s_j must be assigned the same user, if $p_i \neq p_j$, then different users.

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- Generate all non-equivalent complete patterns by enumerating only minimum ones. Natural order, i.e., $(1, 2, 1) < (2, 1, 2)$.

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- Bell numbers have been studied by mathematicians since the 19th century, but their roots go back to medieval Japan.

Basic Algorithm: Part 2

- For each complete pattern p (as partition) $S_1 \uplus \cdots \uplus S_t = S$ construct a bipartite graph B with bipartition $(\{S_1, \ldots, S_t\}, U)$ such that $S_i u \in E(B)$ if u is authorised for all steps in S_i.

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- Poly-space if we check all constraints just after each pattern is generated.

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- Partial pattern: vector $p = (p_1, \ldots, p_k)$, where $0 \leq p_i \leq k$, if $p_i = 0$ then s_i has not been considered yet, if $p_i = p_j > 0$ then s_i, s_j must be assigned the same user, if $0 \neq p_i \neq p_j \neq 0$, then different users. The scope of p is $\{s_i : \ p_i \neq 0\}$.

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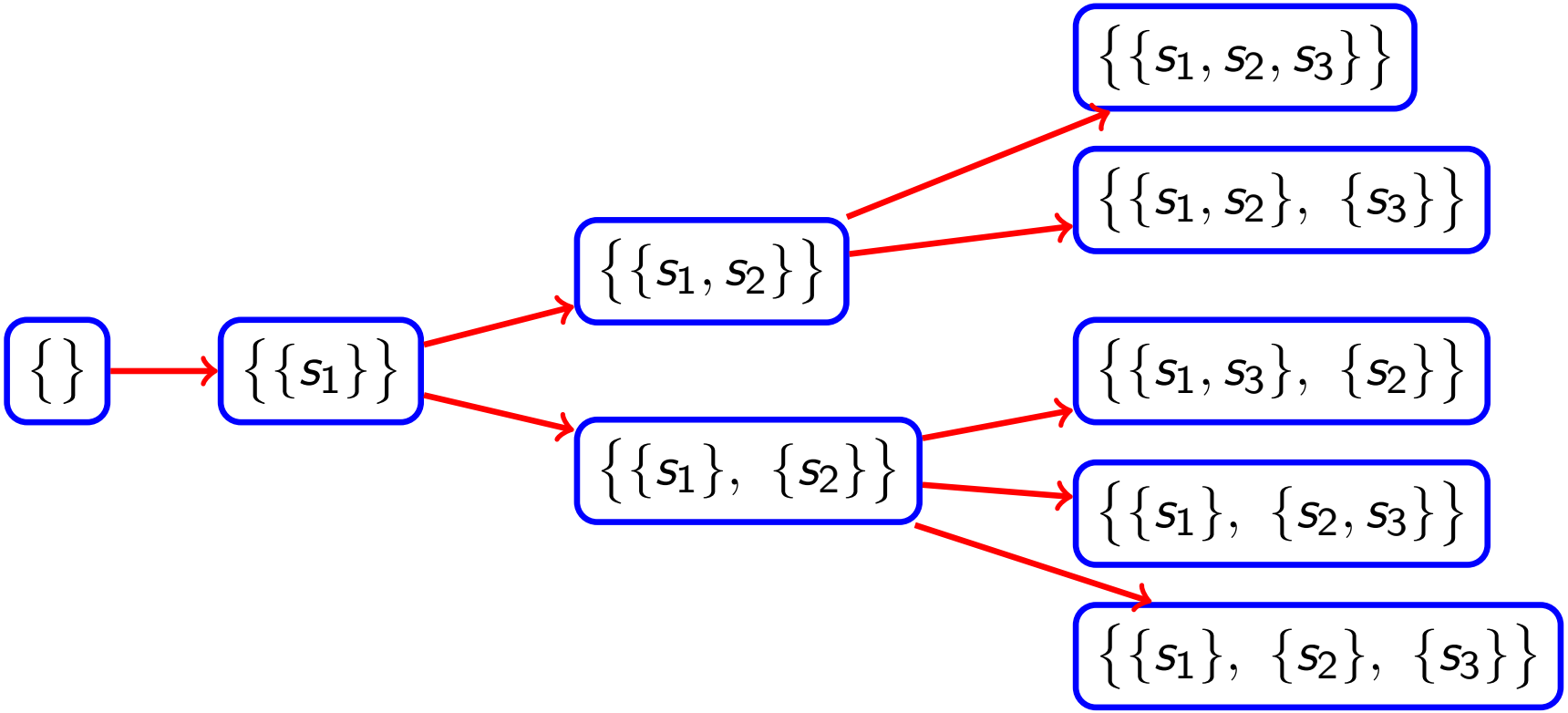
- A special procedure to decide which step to add depending on non-unary constraints.

Generating Complete Patterns



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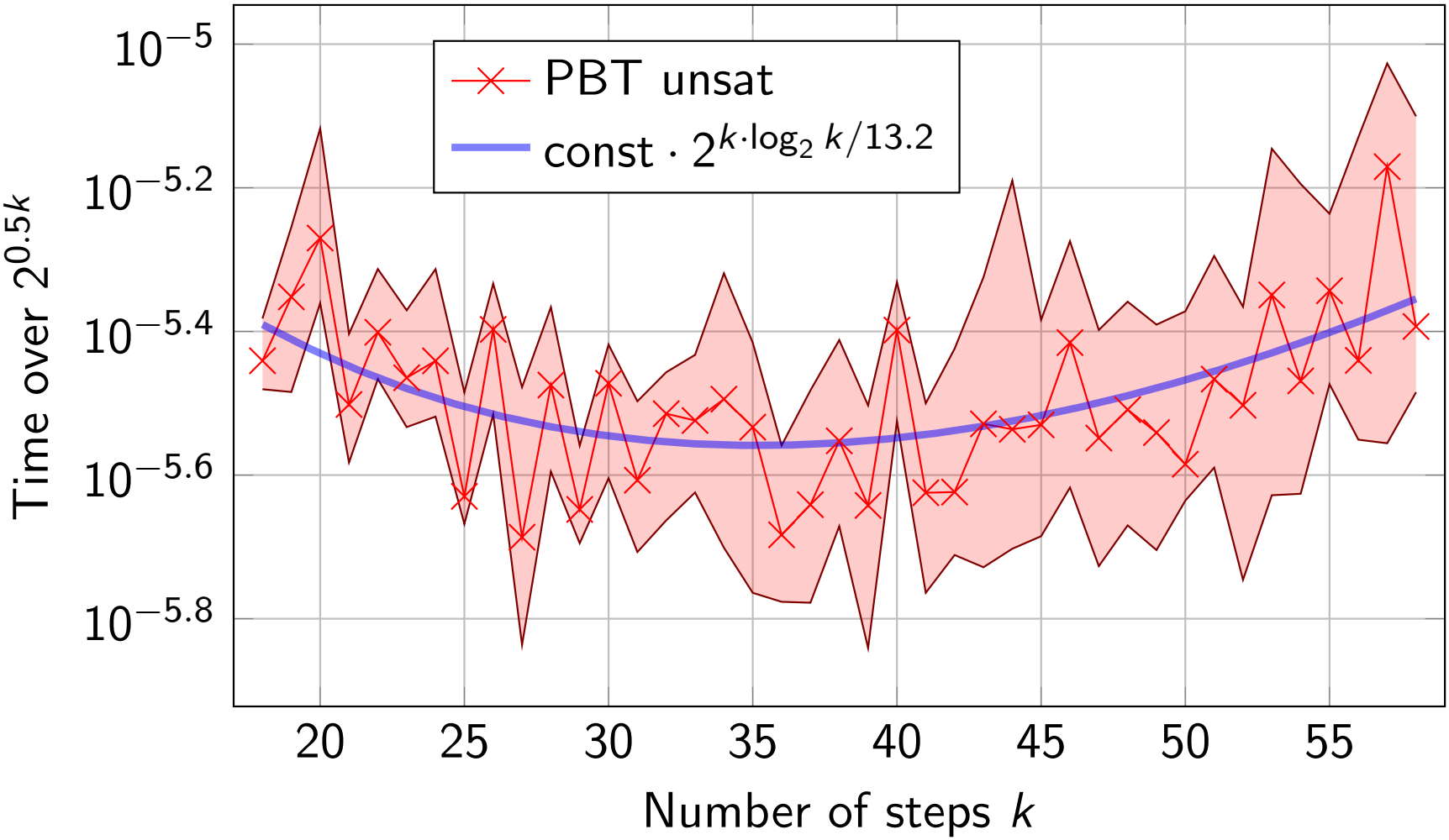
  - e non-equals $c = (s_i, s_j, \neq)$: plan π satisfies c if $\pi(s_i) \neq \pi(s_j)$.
  - e is chosen s.t. $\mathbb{P}[(S, U, C)$ being satisfiable] is approx 50%. They are hardest instances.


PBT unsat
const · $2^{k \cdot \log_2 k/13.2}$
Time over $2^{0.5k}$
10^{-5}
$10^{-5.2}$
$10^{-5.4}$
$10^{-5.6}$
$10^{-5.8}$
20
25
30
35
40
45
50
55
Number of steps k

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CP-SAT CP-SAT from OR-tools (Google) using 'ordinary' CSP formulation of WSP

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- $x_{s,u} = 1$ iff u is assigned to s.
- 'Ordinary' formulation (only not-equals):

$$\begin{aligned} \textstyle\sum_{u \in U} x_{s,u} = 1 \quad & \forall s \in S, \\ x_{s,u} = 0 \quad & \forall s \in S \text{ and } \forall u \in U \setminus A(s), \\ x_{s_1,u} + x_{s_2,u} \leq 1 \quad & \forall \text{ not-equals with scope } \{s_1, s_2\} \text{ and } \forall u \in U. \end{aligned}$$

- 'FPT' formulation uses $x_{s,u}$'s and 'FPT variables' $M_{s,s'}$. $M_{s,s'} = 1$ if s, s' are assigned the same user and $M_{s,s'} = 0$, otherwise. Much longer formulation.

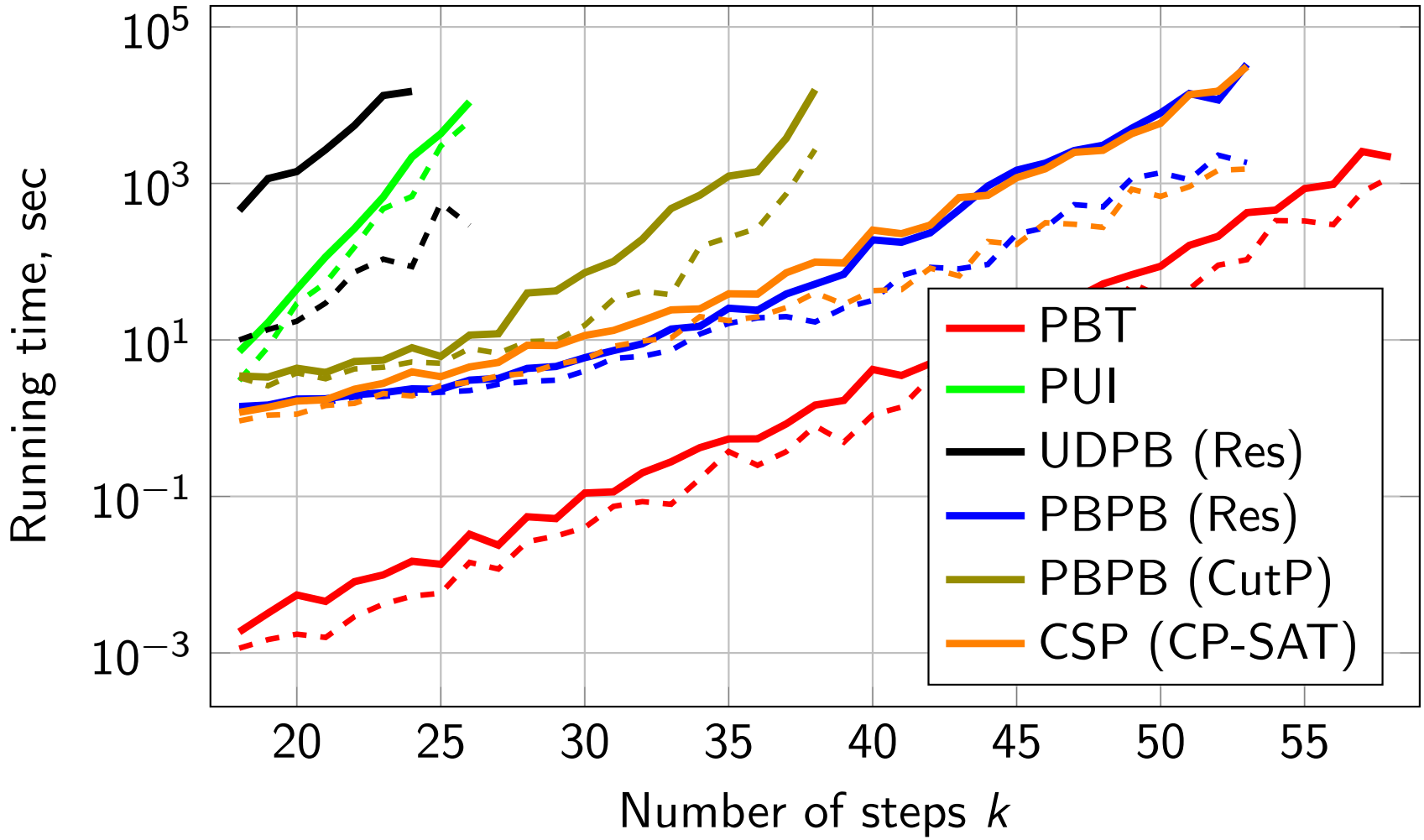
Brief Report of Experimental Results

Theorem

Every UI constraint can be encoded using $M_{s,s'}$'s.

Proof.

Every UI constraint corresponds to a set of patterns $\mathcal{P}$.
Constraints: $M_{s,s'} = M_{s',s}$ for all $s < s'$, $M_{s,s} = 1$ for all $s \in S$, and $\sum_{B \in p} \sum_{s' < s'' \in B} (1 - M_{s',s''}) +$
$\sum_{B' \neq B'' \in p} \sum_{s' \in B'} \sum_{s'' \in B'', s' < s''} M_{s',s''} \geq 1$ for all $p \in \overline{\mathcal{P}}$. □


Running time, sec
10^5
10^3
10^1
10^{-1}
10^{-3}
20
25
30
35
40
45
50
55
Number of steps k
PBT
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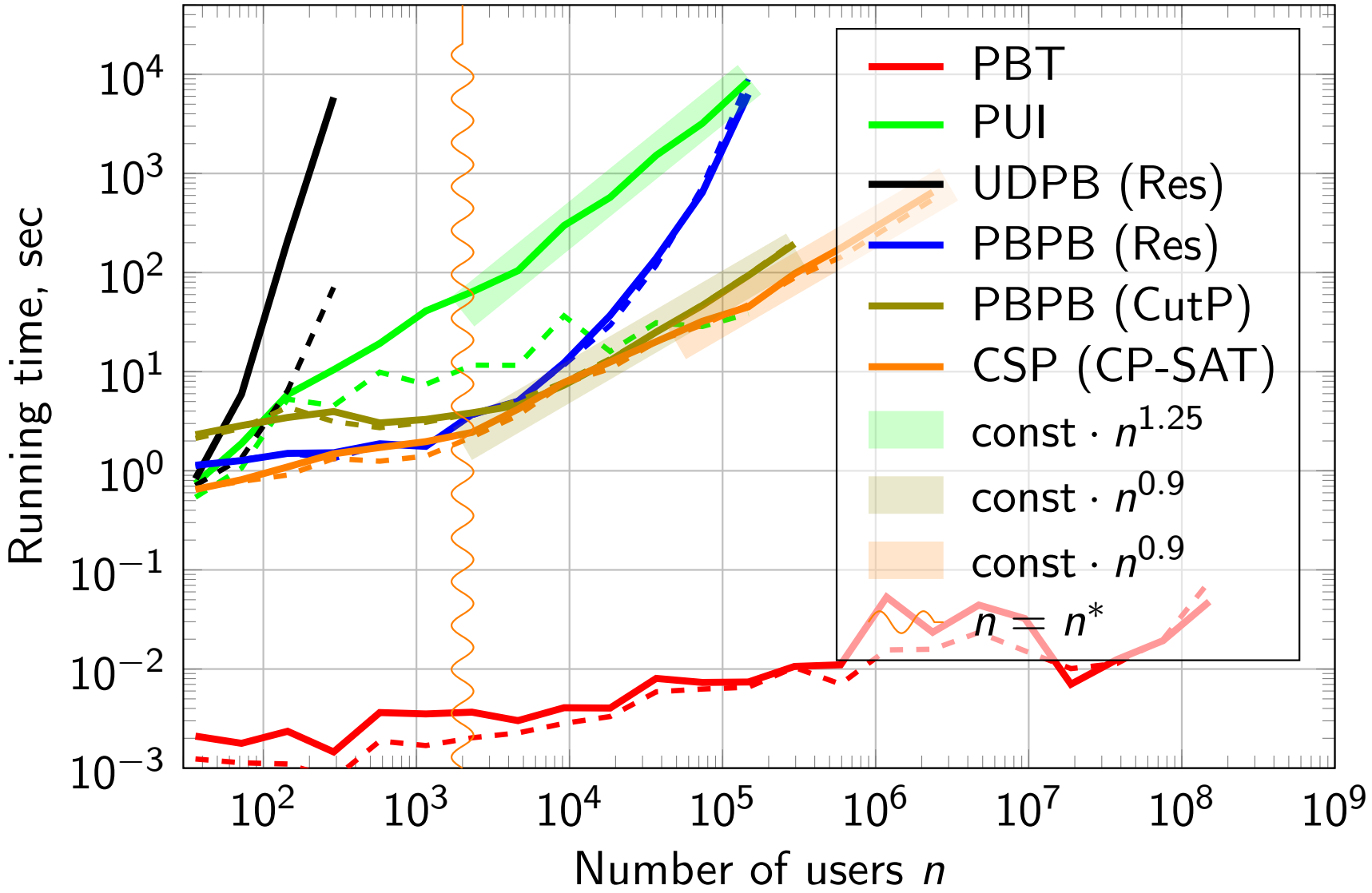
CSP (CP-SAT)


Running time, sec
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^2
10^3
10^4
10^5
10^6
10^7
10^8
10^9
Number of users n
PBT
PUI
UDPB (Res)
PBPB (Res)
PBPB (CutP)
CSP (CP-SAT)
const · $n^{1.25}$
const · $n^{0.9}$
const · $n^{0.9}$
$n = n^*$

Report of Experimental Results

- Winners: PBT, Runners-up: CP-SAT and the best of PBPB (Res) and PBPB (CutP).

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- Likely explanation: CP-SAT formulation is reduced to a SAT formulation. 'FPT variables' appear in the SAT formulation.

Outline

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- If all non-unary constraints are UI, we have an $O^*(2^{k \log k})$-time algorithm for Valued WSP.

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- CPLEX 12.6 solved all instances only for $k = 20$, for $k = 35$ less than 43%.

Thank you! Takk skal du ha!

Any questions?