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Teaching and Supporting the Use of Qualitative and Quantitative Concepts in Classical Mechanics

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Abstract

Though very often quantitative problem solving is accentuated in physics instruction, psychological as well as educational research indicates that this emphasis is misleading. In an experimental study, we compared physics instruction with a focus on quantitative problem solving to physics instruction with a focus on qualitative problem solving. Initially, students were taught quantitative as well as qualitative concepts of classical mechanics by means of concept maps. Thereafter, the students attempted to solve four problems whose solutions demanded the coordinated application of knowledge about quantitative and qualitative concepts. During problem solving, the students received support from tutors. While one group of students was supported in qualitative problem solving, the other group was supported in quantitative problem solving. Before and after the problem solving, the students worked on tests. In accord with our expectations, students who were supported in qualitative problem solving improved significantly more from the pretest to the posttest than students who were supported in quantitative problem solving.

Introduction

Very often, students are not able to successfully approach problems in classical mechanics by means of the knowledge they have acquired during physics instruction. Classical mechanics embodies concepts and relationships between concepts which allow for the description, explanation and prediction of motion. Many concepts and relationships between concepts involve qualitative as well as quantitative information.

Quantitative information is frequently expressed by means of laws which are formalized as algebraic or vector-algebraic equations. Students frequently approach problems which ask for a quantitative solution by only making use of their knowledge about quantitative information. Usually, they start from the variable whose value is in question. Afterwards, they attempt to apply dynamics and kinematics laws in order to determine the variable's value. Very often, however, the students get lost in a muddle of algebraic equations with no means at hand in order to guide their application effectively and efficiently (e.g., Chi, Feltovich & Glaser, 1981; Larkin, 1983).

In contrast to students, experts make use of both their knowledge about qualitative and their knowledge about quantitative information. Initially, they attempt to qualitatively identify the concepts relevant to the problem posed. Subsequently, they take advantage of the qualitative information in order to select the appropriate dynamics and kinematics laws which quantitatively relate the identified concepts to each other (e.g., Chi, Feltovich & Glaser, 1981; Larkin, 1983). Finally, they apply the selected dynamics and kinematics laws in order to determine the value in question.

While experts seem to possess knowledge structures in which knowledge about qualitative and quantitative information is closely related, students' knowledge frequently is not only fragmentary and weakly related but also includes conceptualizations which are inconsistent with the concepts taught during physics instruction (cf. Pfundt & Duit, 1994). Due to these deficiencies, students seem not to be able to take advantage of their knowledge in the same way that experts do. As a consequence, students have to fall back on so-called weak problem solving methods such as operator subgoaling and means-ends analysis (cf. VanLehn, 1996). These methods, however, provide little guidance for solving problems in classical mechanics.

How can students be supported to acquire and to flexibly apply both knowledge about qualitative and quantitative information on classical mechanics? Though very often the emphasis in physics instruction is on quantitative problem solving, this emphasis seems to be misleading (e.g., Hestenes, 1987; Reif & Heller, 1982). Because very often successful quantitative problem solving presupposes qualitative understanding, physics instruction with an emphasis on qualitative problem solving might be more beneficial (e.g., Ploetzner, 1995; White, 1993).

In this paper we present an experimental study in which physics instruction with a focus on quantitative problem solving is compared to physics instruction with a focus on qualitative problem solving. Because psychological research (e.g., Chi, Feltovich & Glaser, 1981; Larkin, 1983), educational research (e.g., Hestenes, 1987; Reif & Heller, 1982) as well as research in artificial intelligence (e.g., de Kleer, 1977) indicate that successful quantitative problem solving presupposes qualitative understanding, we hypothesize that emphasizing qualitative problem solving is more effective than emphasizing quantitative problem solving.

Knowledge about Qualitative and Quantitative Concepts in Classical Mechanics

The application domain is made up of textbook problems which refer to one-dimensional motion with constant acceleration. The knowledge investigated is on qualitative and quantitative information involved in concepts of dynamics (e.g., gravitational and normal force) and kinematics (e.g., displacement, velocity and acceleration).

With respect to qualitative information, the focus is on the conditions under which concepts are applicable, the attributes possessed by concepts and the values which concept attributes might have. For instance, knowledge about the kinetic friction force might comprise the qualitative information that a kinetic friction force acts on a body, whenever a normal force acts on the body and the body is moving on a surface which is not frictionless.

With respect to quantitative information, the emphasis is on dynamics and kinematics laws which are formalized as algebraic or vector-algebraic equations. For example, knowledge about the kinetic friction force might comprise the quantitative information that the magnitude F_f of the kinetic friction force on a body equals the magnitude F_N of the normal force on the same body times the coefficient of friction f: $F_f = F_N \cdot f$.

Qualitative and quantitative information can be conceptualized as complementary information (e.g., de Kleer, 1977). Qualitative information refers to essential features to be taken into account as well as to important distinctions to be drawn. While quantitative information frequently helps to resolve ambiguities inherently involved in qualitative information, the appropriate use of quantitative information very often seems to presuppose the utilization of qualitative information.

Ploetzner (1995) implemented formal representations of qualitative and quantitative information on classical mechanics in a simulation program. If the program is applied to the formal representation of a problem, it simulates how a qualitative problem representation can be taken advantage of to guide the construction of a quantitative problem representation. The program coordinates qualitative and quantitative problem representations in two different ways. Firstly, the information included in a qualitative problem representation is partially transformed into algebraic expressions in order to construct additionally required quantitative information. Secondly, the information contained in a qualitative problem representation is exploited to constrain the use of already available quantitative information.

Method

Design

The study comprised two groups of students and was made up of five sections.

In the first section, all students worked on an introduction to concept maps as well as on an introduction to a computerized concept mapping tool. In the second section, all students studied the same instructional unit which described qualitative and quantitative information on classical mechanics by means of concept maps. In the third section, all students worked on a multi-component test which assessed the knowledge about qualitative and quantitative information the students had acquired during the study of the instructional unit.

In the fourth section, the students attempted to solve four problems which demanded the coordinated use of knowledge about qualitative and quantitative information. During problem solving, the students took advantage of the computerized concept mapping tool. In addition, the students received support from tutors. While one group of students was supported in qualitative problem solving, the other group of students was supported in quantitative problem solving.

Finally, all students worked on a parallel multi-component test which assessed the knowledge about qualitative and quantitative information the students had acquired due to the support from tutors.

Materials

Introduction to Concept Maps To be knowledgeable in a domain means to know the relevant concepts as well as the relationships between them. This structural aspect of knowledge can be represented by means of concept maps (e.g., Jonassen, Beissner & Yacci, 1993). Concept maps form an external representation in which information is structured by means of graphs. Individual nodes represent concepts; the directed and undirected links between the nodes represent relationships between the concepts. In an earlier study, Ploetzner, Fehse, Kneser and Spada (1999) demonstrated that concept maps can be equally well employed to teach qualitative as well as quantitative concepts in classical mechanics.

Because qualitative and quantitative information on classical mechanics were taught to the students by means of concept maps, in the first section of the study, the students worked on an introduction to concept maps in order to learn how concept maps are structured. The concepts addressed in the introduction referred not to classical mechanics but to well-known household furniture.

Computerized Concept Mapping Tool When concept maps are constructed by paper and pencil, they are frequently difficult to extend and to modify. Furthermore, the construction of concept maps can hardly be reconstructed by conventional observation methods. The use of a computerized concept mapping tool, however, allows one to overcome these drawbacks. Therefore, whenever the students had to construct concept maps, they took advantage of such a tool (cf. Ploetzner, Hoppe, Fehse, Nolte & Tewissen, 1996).

In a computerized concept mapping tool, the concepts and relationships relevant to the domain under scrutiny may be made available to the students in advance by means of menus, for example. If needed, the students may fill in additional concepts and relationships at run time. Complete concept maps as well as parts of concept maps may be selected by the mouse and subsequently be moved, copied or deleted. Concept maps are easily re-arranged as well as saved and reloaded. In addition, every step taken to construct, extend or modify a concept map can be saved for later analysis.

In order to learn how to use the computerized concept mapping tool, in the first section of the study, the students worked on an introduction to the tool. As in the introduction to concept maps, the concepts addressed in the introduction to the concept mapping tool referred not to classical mechanics but to well-known household furniture.

Instructional Unit We designed an instructional unit to teach the students qualitative and quantitative information on classical mechanics by means of concept maps. It was made up of three parts. In the first part, coordinate systems and vectors as well as the addition and resolution of vectors were described. In the second part, qualitative and quantitative information on kinematic concepts such as displacement, velocity and acceleration was presented. In the third part, qualitative and quantitative information on dynamic concepts such as gravitational force, normal force, friction force and resultant force was delineated.

The qualitative and quantitative information on the different concepts was described by means of concept maps. One or more concept maps were followed by several examples and exercises. The solutions to the exercises were also presented. In 100 pages total, the unit comprised 30 concept maps, 18 examples and 20 exercises along with their solutions.

The students worked on the instructional unit in the second section of the study. In a first step, they attempted to elaborate the information included in a concept map. In a second step, the students had the opportunity to consider an example. It illustrated the consequences of applying the information included in a concept map to a certain arrangement. In a third step, the students themselves exercised the application of the information included in a concept map to other arrangements. While some of the exercises asked for the construction or completion of diagrams, other exercises asked for the construction of concept maps. The students always constructed diagrams by paper and pencil. Concept maps were always constructed by taking advantage of the computerized concept mapping tool. Finally, the students were allowed to compare their solution to an exercise with the solution presented in the instructional unit.

Problems to be Solved with Support from Tutors Four different problems for problem solving with support from tutors were set up. For example:

> A sledge of mass m = 10 kg moves on a horizontal surface with a velocity of v_0 = 4.8 m/s. The coefficient of friction between the runners of the sledge and the surface equals f = 0.12. After which distance r has the sledge's velocity reduced to v = 0 m/s?

By making use of a simulation program of qualitative and quantitative problem solving in classical mechanics (Ploetzner, 1995), the problems were designed in such a way that – relative to the information presented in the instructional unit – their solutions demanded the coordinated application of knowledge about both qualitative and quantitative information. In order to design the problems, the simulation program was equipped with formal representations of the qualitative and quantitative information which was presented in the instructional unit. Afterwards, the simulation program was applied to formal representations of the four problems. When the simulation program was furnished with either qualitative or quantitative information, its problem solving attempts failed. The problem solving attempts succeed only when the simulation program was furnished with both qualitative and quantitative information.

Strategies Applied by the Tutors In the fourth section of the study, the students attempted to solve the four problems with support from tutors. While one group of students was supported in qualitative problem solving, the other group of students was supported in quantitative problem solving. Two physics students from the School of Education at Freiburg served as tutors. Both were trained to support the students in either qualitative or quantitative problem solving by means of two different problem solving strategies. The strategies are described in Table 1. The strategy to support qualitative problem solving focused on the construction and interpretation of free-body diagrams. The strategy to support quantitative problem solving addressed the systematic use of algebraic equations.

Table 1: The strategies applied by the tutors

Strategy to support qualitative problem solving

1. Drawing a sketch:
   - Identify the body!
   - Is the body in contact with the surface?
   - Draw a sketch!
2. Determining the resultant force:
   - Determine the forces on the body!
   - Draw an arrow for each force!
   - Determine the resultant force on the body!
   - Describe the resultant force algebraically!
   - Is it possible to simplify the algebraic description?
   - Draw a coordinate system!
   - Describe the magnitude of the resultant force relative to the coordinate system!
3. Relating the resultant force to the acceleration:
   - How is the resultant force related to the body's acceleration?
   - Determine the direction of the body's acceleration!
   - Determine the direction of the body's velocity!
   - How does the acceleration affect the velocity?

Strategy to support quantitative problem solving

1. Identifying the given and sought variables:
   - Identify the variables whose values are given!
   - Identify the variables whose values are sought!
2. Selecting an algebraic equation:
   - Select an equation which includes a variable whose value is sought!
   - Attempt to apply Newton's second law $\Sigma F = m \cdot a$!
3. Applying an algebraic equation:
   - Identify the variables whose values are known!
   - Identify the variables whose values are unknown!
   - If the values of all variables in an equation are known except the value which is sought, then substitute the variables for their values and compute the value which is sought!
   - Otherwise, select equations which include variables whose values are unknown and determine the unknown values!
   - After applying an equation, verify the units!

Initially, the tutors explained and demonstrated the problem solving strategy they supported. Thereafter, the students attempted to solve the four problems. They worked on each problem in two phases. In the first phase, the students approached a problem on their own. To describe a problem's solution, the students constructed diagrams by paper and pencil as well as concept maps by taking advantage of the computerized concept mapping tool.

In the second problem solving phase, the students received support from the tutors. The tutors assisted the students after they completed the first problem solving phase or when they did not show any further progress in their problem solving attempts. If the students raised questions which concerned problem solving steps addressed by the tutors' problem solving strategy, the tutors delineated the problem solving steps and encouraged the students to carry them out. If the students were not able to accomplish this, the tutors explained and demonstrated the problem solving steps. Afterwards, the students had to reproduce the tutors' explanation using their own words.

The tutors also encouraged the students to explain their partial or complete solution to a problem. Whenever a problem solving step addressed by the tutors' problem solving strategy was correct, the tutors provided affirmative feedback to the students. Whenever a problem solving step addressed by the tutors' problem solving strategy was incorrect or missing, the tutors indicated the error or omission to the students. Thereafter, the tutors encouraged the students to correct or add the problem solving step. Again, if the students were not able to accomplish this, the tutors explained and demonstrated the step. Afterwards, the students had to reproduce the tutors' explanation using their own words.

Multi-Component Tests In the third as well as in the fifth section of the study, the students worked on a multi-component test which assessed their knowledge about qualitative and quantitative information on classical mechanics. Each test was made up of three different components and comprised 16 problems in total. In order to design the problems, we again took advantage of the simulation program of qualitative and quantitative problem solving in classical mechanics (cf. Ploetzner, 1995).

The first component comprised four problems which assessed knowledge about qualitative information on classical mechanics. These problems were designed in such a way that – relative to the information presented in the instructional unit – their solutions only demanded the application of knowledge about qualitative information on classical mechanics. Correspondingly, the second component comprised four problems which only required the application of knowledge about quantitative information. The third component was made up of eight problems whose solutions demanded the coordinated application of knowledge about both qualitative and quantitative information.

Both tests comprised parallel problems. Each pair of parallel problems were designed in such a way that the same knowledge was applied by the simulation program of qualitative and quantitative problem solving to solve them. However, non-structural features such as the involved entities and numerical values varied across parallel problems. Within each test, the problems were arranged in random order.

The design of the tests allows one to hypothesize which problem solving performance should be observable in the three test components of the pre- and posttest.

With respect to the first test component on qualitative information, we predict that many problems can already be solved in the pretest after studying the instructional unit. While the qualitatively supported students should further improve from the pre- to the posttest, the quantitatively supported students should not do so.

With respect to the second test component on quantitative information, we also hypothesize that many problems can already be solved in the pretest. While the quantitatively supported students should further improve from the pre- to the posttest, the qualitatively supported students should not do so.

In contrast, with respect to the third test component on the coordination of qualitative and quantitative information, we predict that only few problems can already be solved in the pretest. Both qualitatively and quantitatively supported students should improve from the pre- to the posttest. We especially hypothesize, however, that qualitatively supported students improve considerably more than quantitatively supported students.

Subjects

Twenty-four tenth graders, 11 girls and 13 boys, from three different high schools volunteered for the study. While the group of students which was supported in qualitative problem solving comprised 6 girls and 6 boys, the group of students which was supported in quantitative problem solving comprised 5 girls and 7 boys.

Before the study was conducted, the students' general ability was assessed by means of the Advanced Progressive Matrices Test (Raven, 1976). Subsequently, two students who had received the same or almost the same test scores were assigned to different groups. While the average test score of the students who received support in qualitative problem solving was 24.33 (SD = 3.60), the average test score of the students who received support in quantitative problem solving was 23.92 (SD = 3.85). Students from different schools also were equally distributed among the two groups. Furthermore, in each group of students, one half of the students received support from one tutor and the other half received support from the other tutor. The students were paid for their participation.

Because in German high schools Newtonian mechanics is commonly taught to eleventh graders, none of the students had attended classes on Newtonian mechanics as it was addressed in this study.

Procedure

The students were investigated individually for four days running. On the first day, they worked on the introduction to concept maps, on the introduction to the computerized concept mapping tool, and on the first part of the instructional unit. On the second day, the students worked on the remaining parts of the instructional unit and on the pretest. On the third day, the students attempted to solve the first two problems with support from tutors. Finally, on the fourth day, the students attempted to solve the remaining two problems with support from tutors and worked on the posttest.

Results

Times Spent

On average, both groups spent virtually the same amount of time on the different sections of the study (M = 73 vs. M = 75 minutes on the introduction, M = 221 vs. M = 219 minutes on the instructional unit, M = 78 vs. M = 85 minutes on the pretest, M = 154 vs. M = 159 minutes on problem solving and M = 88 vs. M = 86 minutes on the posttest).

Problem Solving Performance

The average relative solution frequencies in the first test component, which assessed knowledge about qualitative information on classical mechanics, are displayed in Figure 1. In accordance with our expectations, the students had acquired considerable knowledge about qualitative information by studying the instructional unit. With respect to the first test component, although statistically not significant, only the qualitatively supported group improved a little from the pretest to the posttest.

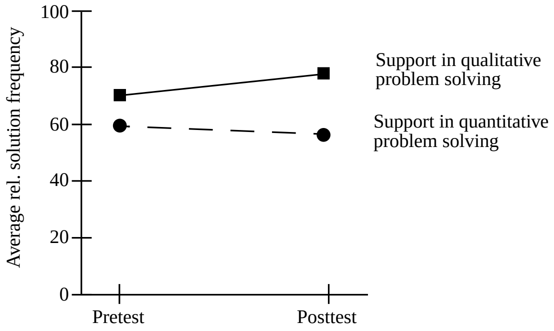

Figure 1: Problem solving performance in the test component on qualitative information.

The average relative solution frequencies in the second test component, which assessed knowledge about quantitative information on classical mechanics, are shown in Figure 2. Again, as expected, the students had acquired substantial knowledge about quantitative information by studying the instructional unit. Furthermore, on average, the qualitatively as well as the quantitatively supported group improved significantly from the pretest to the posttest ($F(1, 22) = 27.72$, $p < .001$).

Figure 3 displays the average relative solution frequencies in the third test component which assessed the coordinated use of knowledge about qualitative and quantitative information on classical mechanics. In accord with our expectations, with respect to this test component, the students exhibited rather poor performance after studying the instructional unit. On average, both groups improved significantly from the pretest to the posttest ($F(1, 22) = 46.48$, $p < .01$). Furthermore, the interaction *Test* x *Group* indicates that the qualitatively supported group improved significantly more from the pretest to the posttest than the quantitatively supported group ($F(1, 22) = 4.47$, $p < .05$).

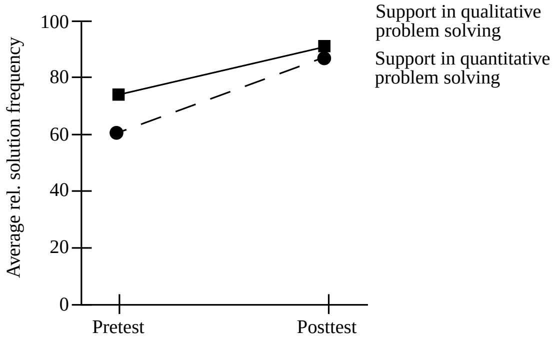

Figure 2: Problem solving performance in the test component on quantitative information.

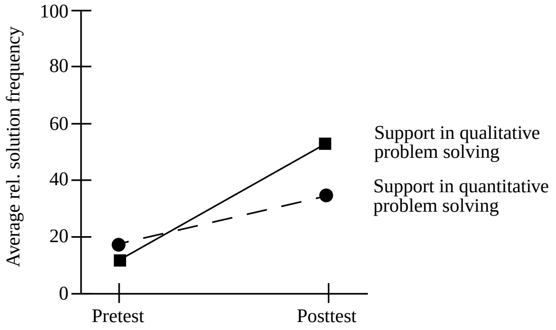

Figure 3: Problem solving performance in the test component on qualitative and quantitative information.

Problem Solving Approach

With respect to the third test component, which assessed the coordinated use of knowledge about qualitative and quantitative information on classical mechanics, it was also analyzed how frequently the students approached these problems qualitatively and quantitatively.

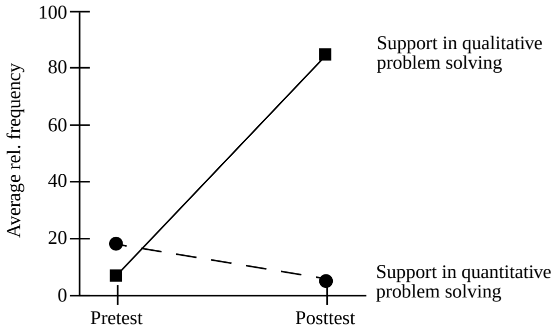

Figure 4: Qualitative problem solving approaches.

The average relative frequencies of qualitative and quantitative problem solving approaches are shown in Figure 4 and 5. The average relative frequency of qualitative problem solving approaches increased significantly from the pretest to the posttest ($F(1, 22) = 54.68$, $p < .01$). Due to the support from tutors, the students who were supported in qualitative problem solving drew more frequently a free-body diagram than the students who were supported in quantitative problem solving ($F(1, 22) = 28.73$, $p < .01$). The interaction *Test* x *Group* further demonstrates the consequences of the support from tutors. While the qualitatively supported group largely increased the number of qualitative problem solving attempts from the pretest to the posttest, the quantitatively supported group even decreased the number of qualitative problem solving attempts ($F(1, 22) = 103.38$, $p < .01$).

The average relative frequency of quantitative problem solving approaches also increased significantly from the pretest to the posttest ($F(1, 22) = 17.75$, $p < .01$). As expected, however, with respect to the use of algebraic equations the qualitatively supported group did not differ significantly from the quantitatively supported group. There is also no statistically significant interaction *Test* x *Group*.

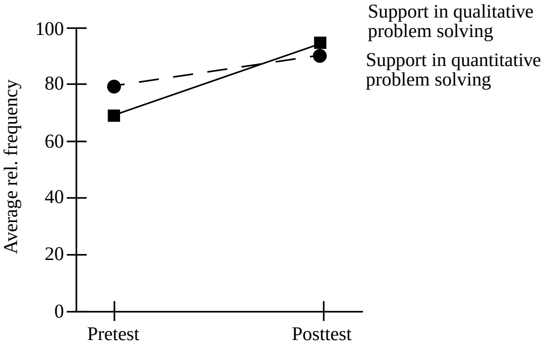

Figure 5: Quantitative problem solving approaches.

Discussion

We presented an experimental study which started from the hypothesis that physics instruction with an emphasis on qualitative problem solving is more effective than physics instruction with an emphasis on quantitative problem solving. The focus of our analysis was on the solution of problems which demand the coordinated application of knowledge about qualitative and quantitative information on classical mechanics.

In such a context, the support of qualitative reasoning as well as the support of quantitative reasoning should enhance the students' problem solving performance. However, while quantitative information frequently helps to guide the use of qualitative information, the appropriate use of quantitative information very often seems to presuppose qualitative understanding (e.g., Chi, Feltovich & Glaser, 1981; de Kleer, 1977; Ploetzner, 1995). Without qualitative understanding, the duality of the physical situation under scrutiny and the quantitative structure set up gets easily lost. Therefore, we expected that the support of qualitative reasoning improves the students' problem solving performance more than the support of quantitative reasoning.

The results are in accord with our expectations. Both the support of qualitative reasoning and the support of quantitative reasoning significantly improved the students' problem solving performance. Especially, students who were supported in qualitative problem solving improved significantly more than students who were supported in quantitative problem solving.

Our results also underline an observation repeatedly made in psychological and educational research on problem solving in formal sciences such as physics. When problems have to be solved which ask for a precise quantitative solution, students strongly tend to focus on the use of quantitative-numerical information and to neglect the use of qualitative-conceptual information. While in the presence of quantitative problems the necessity to make use of quantitative-numerical information seems to be obvious to the students, the necessity of applying qualitative-conceptual information needs again and again to be pointed out to the students as well as its use needs to be encouraged and supported.

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